In-Class Statistics Masters and Ph.D. Qualifying Exam

August 9, 2017, 9:00am-1:00pm

Instructions: *The exam has 7 multi-part problems. All of the problems will be graded. Write your code words on each of your answer sheets. Do not put your name on any of the sheets. Be clear, concise, and complete. All solutions should be rigorously explained.*

Problem 1. (20 pts) X and Y have a joint density given by

$$f(x,y) = \begin{cases} 2, & \text{for} \quad 0 < y < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

(a) If $V = -\ln X$, what is the density of V?

(b) If $V = -\ln X$ and $W = X + Y$, what is the joint density of V and W? Sketch the region for which the joint density is nonzero.

Problem 2. (15 pts) A fair die is tossed 20 times in succession. Let Y be the total number of sixes that occur, and let X be the number of sixes occurring in the first 5 tosses. Determine the conditional probability mass function $P(X = x|Y = y)$.

Problem 3. (15 pts) A bin contains 20 pieces of bread. 16 of the 20 pieces are biscuits, and the remaining 4 pieces are rolls. The 20 pieces of bread are randomly placed into 10 lunchboxes with 2 pieces going into each lunchbox. Let X denote the number of lunchboxes that have 2 biscuits in them. What are the mean and variance of X?

Problem 4. (15 pts) The data observed are -0.4 and 0.8, which are assumed to be realizations of two iid random variables with density

$$f(x|\theta) = \begin{cases} \frac{1}{6} - 2\theta x + x^2, & \text{for } -1 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

(a) Using the data, compute the method of moments estimate of θ.

(b) Find the MLE of θ for these data.

Problem 5. (15 pts) Let $X_1, X_2, \cdots, X_n$ be a random sample from a distribution with pdf

$$f(x|\theta) = \begin{cases} \frac{1}{\theta}, & x \in \{1, 2, \cdots, \theta\}, \theta \in \{1, 2, \cdots\} \\ 0, & \text{elsewhere} \end{cases}$$

(a) Find a minimal sufficient statistic for θ.

(b) Show that the minimal sufficient statistic is complete.

(c) Show that $2\bar{X} - 1$ is an unbiased estimator of θ.

(d) Is $2\bar{X} - 1$ a UMVUE? Why or why not?

Problem 6. (10 pts) Let $X_1, X_2, \cdots, X_n$ be iid $N(0, \sigma^2)$ random variables. Find a uniformly most powerful (UMP) level α test for $H_0 : \sigma^2 \leq \sigma_0^2$ vs. $H_\alpha : \sigma^2 \geq \sigma_0^2$.

Problem 7. (10 pts) Let $X_1, X_2, \ldots, X_{100}$ be a random sample from a distribution with the density function

$$f(x|\theta) = \begin{cases} \frac{2}{\theta^2} x, & 0 < x < \theta \\ 0, & \text{elsewhere} \end{cases}$$

Find an approximate 95% confidence interval for θ.