Prerequisite Tutorial 1 *Exponents and Radicals*

ⓘ Laws of Exponents

ExA

RULES

1. $b^m b^n = b^{m+n}$ and $\frac{b^m}{b^n} = b^{m-n}$

2. $(b^m)^n = b^{mn}$

EXAMPLES

1. $x^5 x^2 = x^7$ and $\frac{x^5}{x^2} = x^3$

2. $(x^5)^2 = x^{10}$

Ⓗ To remember rules 1) and 2), think of ***AMP***... **A**ddition, **M**ultiplication, **P**owers!

ADD exponents when **M**ULTIPLING like bases.

MULTIPLY exponents when raising to a successive **P**OWERS.

A M P

The two operations are consecutive in the order of mathematical magnitude.

RULES

3. $(ab)^n = a^n b^n$ and $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

4. $b^{-n} = \frac{1}{b^n}$

5. $b^{m/n} = \left(b^{\frac{1}{n}}\right)^m$ *or* $(b^m)^{1/n}$

Ⓗ With a fractional exponent, 1st take the "root" (number in bottom of fraction), then the "power."

EXAMPLES

3. $(3x)^4 = 81x^4$ and $\left(\frac{3}{x}\right)^4 = \frac{81}{x^4}$

4. $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

5. $16^{3/2} = \left(16^{\frac{1}{2}}\right)^3 = 4^3 = 64$ *or* $(16^3)^{1/2} = (16^2 16)^{1/2} = 16 \cdot 4 = 64$

CORRECT	INCORRECT
$3^{-2} = 1/3^2$	$3^{-2} \neq 3^{1/2}$
	$3^{-2} \neq -3^2$
$8x^{-2} = \frac{8}{x^2}$	$8x^{-2} \neq \frac{1}{8x^2}$

The exponent is *only* on the *x* and not the 8

More complex example: **ExB**

$$\left(\frac{8x^{-1}}{x^3}\right)^{\frac{2}{3}} = \left(\frac{8}{x^3x^1}\right)^{\frac{2}{3}} = \left(\frac{8}{x^4}\right)^{\frac{2}{3}} = \frac{8^{\frac{2}{3}}}{(x^4)^{2/3}} = \frac{4}{x^{8/3}}$$

ⓘ The negative exponent on top: "Cross the line, change the sign" so it's on the bottom.

💣 Always S! inside before "distributing" outer exponent. Also, remember, to put exponent on ALL factors, including the coefficient.

ⓘ 8 to the 2/3 power is 8 to the 1/3 power (which is 2), squared (which is 4)

Your turn!

1. $\left(\frac{x^3}{27y^{-6}}\right)^{\frac{2}{3}} =$

2. $\left(\frac{4x^2x^4}{y^3}\right)^{-\frac{1}{2}} =$

3. $12^0(s+t)^{-3} =$

ⓘ Laws of Radicals

ExC

RULES

1. $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$ and $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

2. $\sqrt[n]{b^m} = b^{\frac{m}{n}}$

EXAMPLES

1. $\sqrt[3]{2x} \cdot \sqrt[3]{4x^5} = \sqrt[3]{8x^6}$
$= 2x^2$
and $\frac{\sqrt[3]{32x^4}}{\sqrt[3]{4}} = \sqrt[3]{\frac{32x^4}{4}}$
$= \sqrt[3]{8x^3x}$
$= 2x\sqrt[3]{x}$

2. $\sqrt[4]{16^3} = 16^{\frac{3}{4}}$
$= 2^3$
$= 8$

Multiplication Properties that do *not* work with addition!

$(xy)^2 = x^2y^2$	*BUT*	$(x+y)^2 \neq x^2 + y^2$
$\sqrt{xy} = \sqrt{x}\sqrt{y}$	*BUT*	$\sqrt{x+y} \neq \sqrt{x} + \sqrt{y}$
$\sqrt{x^2y^2} = xy$	*BUT*	$\sqrt{x^2+y^2} \neq x + y$

You can only ADD radicals if the radicands (thing under radical) are the same!

$\sqrt[n]{x^n + y^n} \neq x + y$ just like $(x+y)^n \neq x^n + y^n$

i.e. $\sqrt{x^2+16} \neq x + 4$ just like $(x+4)^2 \neq x^2 + 16$

ExD Rationalizing numerators and denominators:

1. $\frac{5x^2}{\sqrt{3x}}$

$= \frac{5x^2}{\sqrt{3x}} \cdot \frac{\sqrt{3x}}{\sqrt{3x}}$

$= \frac{5x^2\sqrt{3x}}{3x}$

$= \frac{5x\sqrt{3x}}{3}$

If there is more than one term in the radical expression, then the idea is to multiply top and bottom of the fraction by the CONJUGATE of either the top or bottom, depending on whether you are rationalizing the top or the bottom.

2. $\frac{\sqrt{x+3}-\sqrt{x}}{3} = \frac{\sqrt{x+3}-\sqrt{x}}{3} \cdot \frac{\sqrt{x+3}+\sqrt{x}}{\sqrt{x+3}+\sqrt{x}}$

$= \frac{(\sqrt{x+3}-\sqrt{x})(\sqrt{x+3}+\sqrt{x})}{3(\sqrt{x+3}+\sqrt{x})}$

$= \frac{x+3+\sqrt{x}\sqrt{x+3}-\sqrt{x}\sqrt{x+3}-x}{3(\sqrt{x+3}+\sqrt{x})}$

$= \frac{3}{3(\sqrt{x+3}+\sqrt{x})}$

$= \frac{1}{\sqrt{x+3}+\sqrt{x}}$

Your turn!

4. Simplify $\frac{2\sqrt{x^5y^6}}{\sqrt{16xy^3}}$

5. Rationalize the denominator: $\frac{2\sqrt{x}+\sqrt{y}}{2\sqrt{x}-\sqrt{y}}$

Prerequisite Review problems

FYI: You will be required to show your work in the same manner as shown in the previous examples. Be sure to read the HW Guidelines *carefully* before you begin the PR's.

PT1#1 Simplify. Answers should contain no negative exponents.

(a) $8(-8)^{\frac{2}{3}}$ (b) $-32^{\frac{2}{5}}$ (c) $\left(\frac{2x^4y^{-2}}{x^2y^3}\right)^{-3}$ (d) $x\sqrt{16x^3}+\sqrt{x^5}$ (e) $\sqrt[3]{2a^2b}\cdot\sqrt[3]{32a^4b^2}$

***PT1#2** (a) Rationalize the numerator of $\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$ (b) Rationalize the denominator of $\frac{2}{\sqrt{x+2}+\sqrt{x}}$

Prerequisite Review problem answers

PT1#1 (a) 32 (b) -4 (c) $\frac{y^{15}}{8x^6}$ (d) $5x^2\sqrt{x}$ (e) $4a^2b$

PT1#2 (a) $\frac{x-y}{x+2\sqrt{xy}+y}$ (b) $\sqrt{x+2}-\sqrt{x}$

Your Turn answers

1. $\frac{x^2y^4}{9}$
2. $\frac{y^{3/2}}{2x^3}$
3. $\frac{1}{(s+t)^3}$
4. $\frac{x^2y\sqrt{y}}{4}$
5. $\frac{4x+4\sqrt{xy}+y}{4x-y}$