Prerequisite Tutorial 2 *Factoring*

Notation used:
F! Factor
S! Simplify
M! Multiply
GCF Greatest Common Factor
PT1 Prerequisite Tutorial 1

ⓘFactoring

ExA

1) Always factor (**F!**) out the GCF if possible: (a) $3x(x+9)^2+6x^2(x+9)=3x(x+9)[(x+9)+2x]$

GCF: $3x\,(x+9)$

S! what's left behind: combine x terms.

$=3x(x+9)(3x+9)$
$=3x(x+9)3(x+3)$
$=9x(x+9)(x+3)$

F! out a 3 and M! it with the $3x$ in front.

2) Difference of perfect squares: $A^2-B^2=(A-B)(A+B)$ (a) $x^2-4=(x-2)(x+2)$

(b) $9x^2-16y^2=(3x-4y)(3x+4y)$

(c) $3x^7-27x^3=3x^3(x^4-9)$
$=3x^3(x^2-3)(x^2+3)$

3) Difference/Sum of perfect cubes: $A^3-B^3=(A-B)(A^2+AB+B^2)$

$A^3+B^3=(A+B)(A^2-AB+B^2)$

(a) $27y^6-8=(3y^2)^3-2^3$
$=(3y^2-2)(9y^4+6y^2+2)$

4) Factor as a quadratic – the "Anti-FOIL":

1. If it factors, it will factor into 2 binomials.
I put minus signs just as place holders.

2. In the **F**irst spot of each factor has to be 2 things that multiply to give us the 1st term, $2x^2$. There is no choice – it's $2x$ and x.

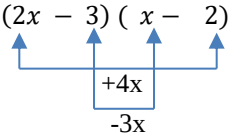
3. In the **L**ast spot of each factor are 2 things that multiply to give us the last term, 6 (don't worry about the negative sign yet). The choices are 6 and 1, or 3 and 2. I always try the pair that is closest together in value first, so 3 and 2. Put them in the blanks and check your outer and inner terms – you need to be able to add or subtract them to get a +1x in the middle. The outer is 4x and the inner is 3x. If the 4x is positive and the 3x is negative, then we get a +1x. So, we change the negative sign on the 2 to a positive.

(a) $2x^2+x-6$

(-)(-)

$(2x-\ \ )(x-\ \ )$

$(2x-3)(x-2)$



$(2x-3)(x+2)$

(b) $x^4-3x^3-4x^2=x^2(x^2-3x-4)$
$=x^2(x-4)(x+1)$

Factor out the GCF of x^2 first.

(c) $x^4-3x^2-4=(x^2-4)(x^2+1)$
$=(x-2)(x+2)(x^2+1)$

Quadratic in terms of x^2

Your Turn!
Perform the indicated operations & simplify.

1. Factor (a) $3(x+1)^2-9(x+1)$ (b) $2x^2(y+1)^2-10x^4(y+1)$ (c) $1-64x^2$ (d) $4x^6y-64x^2y$

(e) $1-64x^3$ (f) $6x^4-4x^3-2x^2$ (g) $12x^2-3y^2$

More complex examples: **ExB**

When factoring out factors with exponents, you always factor out the factor raised to the SMALLEST exponent.

1. Factor: $5x^2(3x^2+1)^4(6x)+(3x^2+1)^5(2x)$

$= 2x(3x^2+1)^4[5x^2(3)+(3x^2+1)]$

$= 2x(3x^2+1)^4[15x^2+3x^2+1]$

$= 2x(3x^2+1)^4(18x^2+1)$

2. Factor: (a) $3x^{2/3}-2x^{1/3}$

$= x^{1/3}(3x^{1/3}-2)$

(b) $3(x-1)^{-2/3}+2(x-1)^{1/3}$

$= 3(x-1)^{-2/3}+2(x-1)^{-2/3}(x-1)^{3/3}$

$= (x-1)^{-2/3}[3+2(x-1)^{3/3}]$

$= \frac{3+2(x-1)}{(x-1)^{2/3}}$

$= \frac{2x+1}{(x-1)^{2/3}}$

Your turn! 2. Factor and simplify

(a) $(1+x^2)^3(2)-2x(2)(1+x^2)^2(2x)$

(b) $2x(x+3)^{2/3}-6(x+3)^{-1/3}$

(c) $2x^2(x+1)^{1/2}-6x(x+1)^{-1/2}$

NOTES

1. $3x^2-6x^5 = 3x^2(1-2x^3)$ We factor out x^2 because 2 is smaller than the 5 on $6x^5$.

2. $3x^{2/3}-6x^{5/3} = 3x^{2/3}(1-2x^{3/3})$
$= 3x^{2/3}(1-2x)$

2/3 + 3/3 = 5/3

We factor out $x^{2/3}$ because 2/3 is smaller than the 5/3 on $6x^{5/3}$.

This is 3/3 because if you think about distributing the $3x^{2/3}$back through you would be multiplying like bases, so adding the exponents, and 2/3 + 3/3 = 5/3

3. $3x^{-2/3}-6x^{1/3} = 3x^{-2/3}(1-2x^{3/3})$
$= 3x^{-2/3}(1-2x)$

We factor out $x^{-2/3}$ because -2/3 is smaller than the 1/3 on $6x^{1/3}$.

This is 3/3 because if you think about distributing the $3x^{-2/3}$back through you would be multiplying like bases, so adding the exponents, and -2/3 + 3/3 = 1/3

Prerequisite Review problems

FYI: You will be required to show your work in the same manner as shown in this tutorial. Be sure to read the HW Guidelines *carefully*.

Your Turn answers

1. (a) $3(x+1)(x-2)$ (b) $2x^2(y+1)(y+1-5x^2)$ (c) $(1-8x)(1+8x)$ (d) $4x^2y(x-2)(x+2)(x^2+4)$

(e) $(1-4x)(1+4x+16x^2)$ (f) $2x^2(3x+1)(x-1)$ (g) $3(2x-y)(2x+y)$

2. (a) $2(1+x^2)^2(1-3x^2)$ (b) $2(x+3)^{-\frac{1}{3}}(x^2+3x-3)$ or $\frac{2(x^2+3x-3)}{(x+3)^{\frac{1}{3}}}$ (c) $\frac{2x(x^2+x-3)}{(x+1)^{1/2}}$