

Prerequisite Tutorial 3 Fractions & Rational Expressions

① Properties of Fractions

ExA (a) $a\left(\frac{c}{b}\right) = \frac{ac}{b}$ (b) $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$ (c) $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$ (d) $\frac{a}{mb} + \frac{c}{md} = \frac{ad+cb}{mbd}$

⚡ Common ERROR: $a\left(\frac{c}{b}\right) \neq \frac{ac}{ab}$

Use the **LCD**! You do NOT want to do $\frac{a}{mb} + \frac{c}{md} = \frac{amd+cmb}{m^2bd}$

(e) $\frac{1}{b} \cdot \frac{1}{c} = \frac{1}{bc}$ **B**
U
T $\frac{1}{b} + \frac{1}{c} \neq \frac{1}{b+c}$ (f) $\frac{ab}{b} = a$ **B**
U
T $\frac{a+b}{b} \neq a$ (g) $\frac{a}{cb^{-1}} = \frac{ab}{c}$ **B**
U
T $\frac{a}{c+b^{-1}} \neq \frac{a+b}{c}$

① Reducing Fractions

Factor & reduce:

Proper reducing:

$$\frac{2x(x+1)}{x+1} = 2x$$

CORRECT

$$\frac{2x-(x+1)}{(x+1)} = \frac{2x-x-1}{(x+1)} = \frac{x-1}{x+1}$$

INCORRECT!

$$\frac{2x-(x+1)}{(x+1)} \neq 2x$$

NO!

⚡ Only common FACTORS reduce!

ExB

ExC Always **FACTOR** and then reduce (R!). Remember, **only common FACTORS cancel**.

$$1) \frac{8x^2 - 6x}{6x^3} = \frac{2x(4x-3)}{6x^3} = \frac{4x-3}{3x^2}$$

The 6x on top does NOT reduce with the 6x on the bottom because they are not common FACTORS!

$$2) \frac{3x(x+1)^2 - x^2(x+1)}{x(x+1)^3} = \frac{x(x+1)[3(x+1) - x]}{x(x+1)^3} = \frac{3x+3-x}{(x+1)^2} = \frac{2x+3}{(x+1)^2}$$

Ⓜ Factor out the G

R! common factors

Your turn!

Use the idea of factoring out the GCF and then reducing.

1. Simplify: (a) $\frac{2a^2-3ab-9b^2}{2ab^2+3b^3}$

(b) $\frac{2(1+x^2)^2-8x^2(1+x^2)}{(1+x^2)^4}$

① Multiplying/Dividing Fractions

ExD To **M!** 2 fractions, always **F!** 1st, **R!** 2nd, and *then* **M!** straight across.

$$1) \frac{2}{9} \cdot \frac{21}{8} = \frac{1}{3} \cdot \frac{7}{4} \\ = \frac{7}{12}$$

H Always REDUCE (R!) before multiplying (M!)
R! the 21 w/the 9 & the 2 w/the 8. Do NOT multiply across to get 42/72 and *then* reduce.

$$2) \frac{x^2 - 4}{x^2 - 4x + 4} \div \frac{x + 2}{2x - 4} = \frac{x^2 - 4}{x^2 - 4x + 4} \cdot \frac{2x - 4}{x + 2} \\ = \frac{(x - 2)(x + 2)}{(x - 2)(x - 2)} \cdot \frac{2(x - 2)}{x + 2} \\ = 2$$

① Adding Fractions

ExE To add 2 fractions, you **MUST** have the same denominator. Be sure to find the **LOWEST** common denominator (LCD).

$$1) \frac{1}{6} + \frac{4}{15} = \frac{1}{2 \cdot 3} + \frac{4}{5 \cdot 3} \\ = \frac{1}{2 \cdot 3} \cdot \frac{5}{5} + \frac{4}{5 \cdot 3} \cdot \frac{2}{2} \\ = \frac{5}{30} + \frac{8}{30} \\ = \frac{13}{30}$$

It's important to note that if both denominators already share a common factor, you only use it **ONCE** in the LCD. This is why you should not always merely multiply the 2 original denominators together to create your common denominator. For example, resist doing the following:

The only way to create the same denominator in both fractions is to **MULTIPLY** both top and bottom of each fraction by the appropriate number. (You are simply multiplying the entire fraction by 1, and thus not changing its value).

$$\frac{1}{6} + \frac{4}{15} = \frac{15}{6 \cdot 15} + \frac{24}{15 \cdot 6} \\ = \frac{15 + 24}{90} \\ = \frac{39}{90} \\ = \frac{13}{30}$$

NO!

Now you have to reduce by that shared factor of 3

$$2) \frac{1}{x^2 + x} - \frac{2}{x^2 - 1} = \frac{1}{x(x+1)} - \frac{2}{(x-1)(x+1)} \\ = \frac{(x-1) - 2x}{x(x+1)(x-1)} \\ = \frac{-x-1}{x(x+1)(x-1)} \\ = \frac{-(x+1)}{x(x+1)(x-1)} \\ = \frac{-1}{x(x-1)}$$

ALWAYS factor denominators 1st. Then find **LOWEST** common denominator. Do NOT use all factors!

ALWAYS ALWAYS ALWAYS leave your denominators **FACTORED**

$$3) \frac{x}{5x+2} + \frac{3}{5x} = \frac{x}{5x+2} \cdot \frac{5x}{5x} + \frac{3}{5x} \cdot \frac{5x+2}{5x+2} \\ = \frac{5x^2 + 15x + 6}{5x(5x+2)}$$

NO!

$$\frac{3}{5x} \neq \frac{3+2}{5x+2}$$

You cannot *add* the same number to the top and bottom of a fraction!

Your turn!

2. Simplify: (a) $\frac{x}{x+1} + \frac{2}{x^2+4x+3}$

(b) $\frac{3}{x^2-1} - \frac{2}{1-x} + \frac{1}{x}$ (Hint: change the $1-x$ to $-(x-1)$)

① Simplifying Fractions

ExF

"Invert & Multiply"

$$1) \frac{4}{1/x} = 4 \cdot \frac{x}{1} = 4x$$

$$2) \frac{4/5}{x} = \frac{4}{5} \cdot \frac{1}{x} = \frac{4}{5x}$$

ExG

A simplified fraction is a REDUCED fraction containing a SINGLE fraction bar.

$$1) \frac{8 + \frac{1}{x}}{16 - \frac{2}{x}} = \frac{8 + \frac{1}{x}}{16 - \frac{2}{x}} \cdot \frac{x}{x} = \frac{8x + 1}{16x - 2}$$

"Trick of the Trade"
A very useful method is to multiply the top and bottom of the overall fraction by the LCD of the interior denominators.

$$\frac{\frac{3}{h+1} - 3}{h} = \frac{3 - 3(h+1)}{h(h+1)} = \frac{3 - 3h - 3}{h(h+1)} = \frac{-3h}{h(h+1)} = \frac{-3h}{h} \cdot \frac{1}{h+1} = \frac{-3}{h+1}$$

Another method is to combine the 2 terms in the numerator, and then "invert and multiply."

$$2) \frac{\frac{3}{h+1} - 3}{h} = \frac{\frac{3}{h+1} - 3}{h} \cdot \frac{h+1}{h+1} = \frac{3 - 3(h+1)}{h(h+1)} = \frac{3 - 3h - 3}{h(h+1)} = \frac{-3h}{h(h+1)} = \frac{-3}{h+1}$$

$$3) \frac{1 + x^{-1}}{1 - x^{-2}} = \frac{1 + \frac{1}{x}}{1 - \frac{1}{x^2}} \cdot \frac{x^2}{x^2} = \frac{x^2 + x}{x^2 - 1} = \frac{x(x+1)}{(x-1)(x+1)} = \frac{x}{x-1}$$

⚠ You can NOT "cross the line" of the overall fraction bar with the x^{-1} or the x^{-2} because they are TERMS and not FACTORS of the top and bottom of the fractions!

$$4) \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \frac{\frac{x - (x+h)}{x(x+h)}}{h} = \frac{\frac{-h}{x(x+h)}}{h} = \frac{-h}{x(x+h)} \cdot \frac{1}{h} = \frac{-1}{x(x+h)}$$

OR... We could have multiplied top and bottom of main fraction by $x(x+h)$ to clear away the "inner" fractions.

Your turn!

3. Simplify: (a) $\frac{\frac{1}{x} + \frac{1}{y}}{1 - \frac{1}{xy}}$ (b) $\frac{\frac{1}{3+h} - \frac{1}{3}}{h}$

Prerequisite Review problems

FYI: You will be required to show your work in the same manner as shown in this tutorial.
Be sure to read the HW Guidelines *carefully*.

Be sure to read/study **ExA-D** before trying **PT3#1!**

PT3#1 Simplify. (a) $3\left(\frac{2}{5}\right)$ (b) $\frac{5}{6} \cdot \frac{4}{15}$ (c) $\frac{2x^2+8x}{8x}$
 (d) $\frac{8x(x+2)^2-6x^2(x+2)}{6x^3(x+2)^6}$ (e) $\frac{x^2-3x}{x^2-4} \div \frac{2x}{x+2}$

PT3#1 (a) $\frac{6}{5}$ (b) $\frac{2}{9}$ (c) $\frac{x+4}{4}$
 (d) $\frac{x+8}{3x^2(x+2)^5}$ (e) $\frac{x-3}{2(x-2)}$

Be sure to read/study **ExE** before trying **PT3#2!**

PT3#2 Simplify and determine domain if applicable.
 (a) $\frac{17}{20} - \frac{7}{8}$ (b) $\frac{2}{5x+1} + \frac{3}{5x}$ (c) $\frac{x^2-3}{x^2-x} - \frac{x+2}{2x}$

PT3#2 (a) $-\frac{1}{40}$
 (b) $\frac{25x+3}{5x(5x+1)}$
 (c) $\frac{x^2-x-4}{2x(x-1)}$

☛ Leave your denominators **FACTORED!!**

Be sure to read/study **ExF-G** before trying **PT3#3!**

PT3#3 Simplify.
 (a) $\frac{48}{8/3}$ (b) $\frac{3/20}{15}$ (c) $\frac{1}{1+\frac{3}{h+1}}$ (d) $\frac{\frac{1}{2+x} \cdot \frac{1}{2}}{x}$

PT3#3 (a) 18 (b) $\frac{1}{100}$ (c) $\frac{h+1}{h+4}$ (d) $\frac{-1}{2(2+x)}$

Your Turn answers

1. (a) $\frac{a-3b}{b^2}$ (b) $\frac{2(1-3x^2)}{(1+x^2)^3}$
 2. (a) $\frac{x+2}{x-3}$ (b) $\frac{3x^2+5x-1}{x(x-1)(x+1)}$
 3. (a) $\frac{y+x}{xy-1}$ (b) $\frac{-1}{3(3+h)}$