Social Structures and Speeding Trucks: A Simulation with Experimentation

The meaning of the computer presence in people's lives is very different from what most expected in the late 1970s. One way to describe what has happened is to say that we are moving from a modernist culture of calculation toward a postmodernist culture of simulation. —Sherry Turkle (1997:349)

Another way to describe what has happened is to say that, in the best of all possible virtual worlds, the most meaningful experiences *combine* calculation and simulation.

Reading Ouellet (1994), we learn that the typical trucker logs 60 to 70 hours per week; that he works "at a steady clip" whether paid by the hour or by the load; that there are subtle pressures that lead to excessive speed, e.g., "drivers often jockey one another for position on the road and by the loading dock, as a minute or two lead to the next job ... can save an hour or two of waiting time." All of which leads us to seek circumstances under which a speed in excess of 65 to 70 miles per hour would appear to be rational. Here, rational means cost-minimizing, or profit-maximizing. How would law enforcement attempt to manipulate these circumstances?[note 1]

Notice that the user, early in this simulation, may experiment with the first four input values, $k1$, x_1, $k3$, $k4$. Later on, we encounter additional opportunities for experimentation based on further theoretical considerations.

```
> restart; job := time():
```

1 Structure

First, we insert a baseline operating cost, in dollars per mile, for a given type of truck. By baseline, we mean all costs involved in paying for and maintaining a truck, worked out on a per-mile basis, but without considering ways in which costs are raised by driver behavior, e.g., high operating speeds, mechanical abuse, etc., or by traffic citations.

Standard cost per mile:

```
> k1 := 1/3;
```

$$k1 := \frac{1}{3}$$

Truck operating costs increase with average speed, due to increased fuel consumption and increased depreciation. We estimate this additional per/mile cost to be a certain multiplier of a dollar: the speed in m.p.h., called v, which we will first adjust with an exponent and then divide by, say, 1000 or some other large standard. This latter value itself may need to be adjusted from time to time to reflect changing costs of fuel, depreciation, etc. I propose that we begin with an equation such as that given immediately below as a plausible way of capturing this variable. The expression containing v could be $\frac{v}{455}$ or $\frac{v^{(\frac{4}{3})}}{2450}$ or $\frac{v^2}{5000}$ and so forth.

```
> x[1] := (v^(8/5))/(4 * (10^3)); # additional velocity factor
```

$$x_1 := \frac{1}{4000} v^{8/5}$$

The next several expressions should be fairly clear. They, too, are subject to change. We consider the situation of drivers who have an hourly pay rate; such drivers, according to Ouellet, are in a very different situation from those who receive piece-work payment.

The hourly wage, initialized below, assumes that for at least part of a lengthy trip, two drivers must be paid, so that the value assigned to $k3$ is a weighted average.

```
> k3 := 3715/100; # driver's hourly wage
```

$$k3 := \frac{743}{20}$$

Trip length is 600 miles.

```
> k4 := 600; # trip length, miles
```

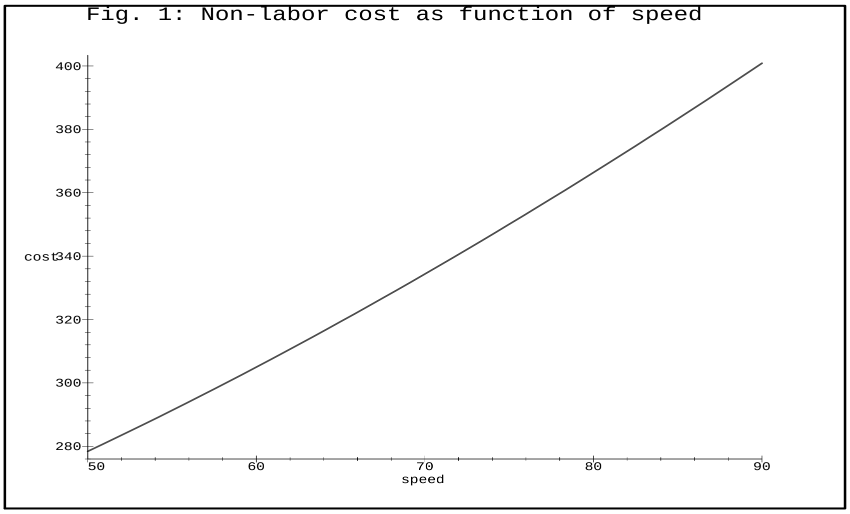
$$k4 := 600$$

Now we obtain a provisional total cost, for trips of this type. The total cost has two components, as in a Cobb-Douglas function (Dowling 1990:213-14): capital cost and labor cost. The first component is the length of the trip multiplied by the sum of $k1$ and x_1:

```
> cn := k4*(k1 + x[1]); # non-labor cost, entire trip
```

$$cn := 200 + \frac{3}{20} v^{8/5}$$

```
> plot(cn, v=50..90, thickness = 3, labels = [speed, cost], title
= `Fig.  1:  Non-labor cost as function of speed`);
```

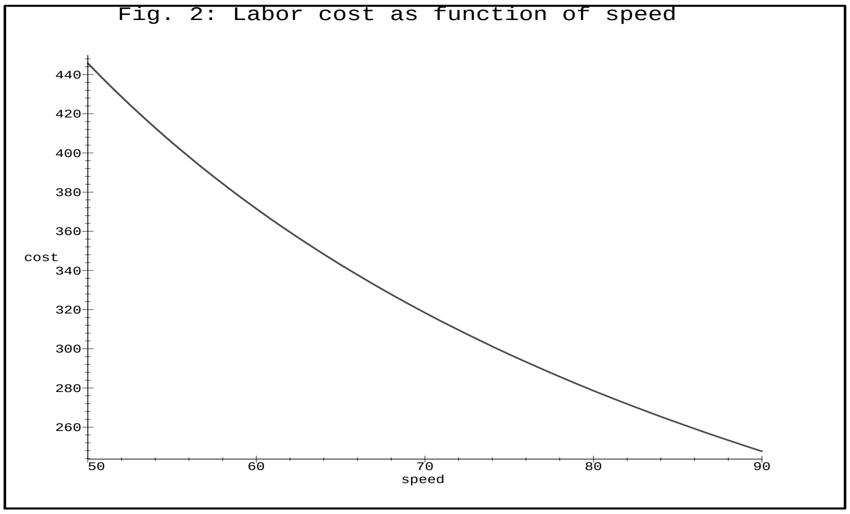


The second cost component is the driver's hourly wage multiplied by the amount of time required for the trip, which is the trip length divided by the speed: $cl = k3\,\frac{k4}{v}$

```
>  cl := k3*(k4/v); # labor cost, entire trip
```

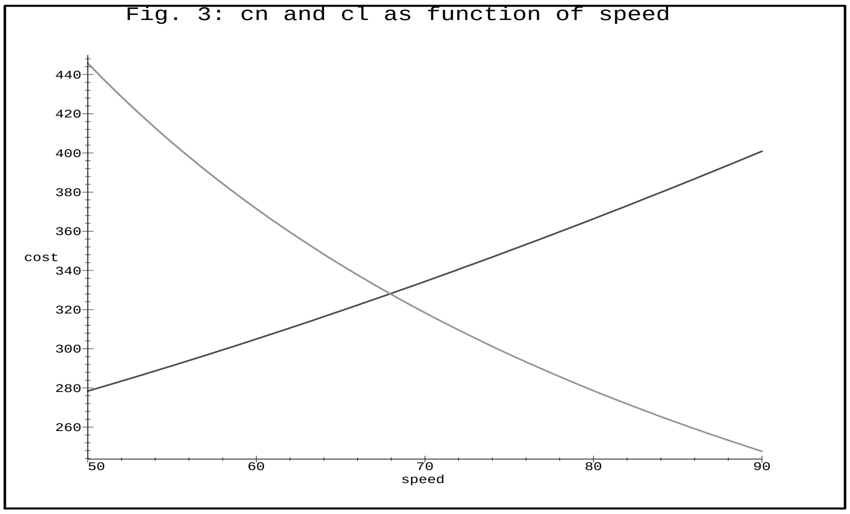
$$cl := \frac{22290}{v}$$

```
>  plot(cl, v=50..90, thickness = 3, labels = [speed, cost], title
= `Fig.  2:  Labor cost as function of speed`);
```



Here we have an opportunity to examine non-labor costs and labor costs together, as functions of speed, on a single plot. When we set out to find the speed that minimizes costs, we shall realize that this is a speed for which non-labor costs are *increasing* at the same rate as labor costs are *decreasing*. One could readily obtain derivatives for these two functions separately, and see how they "cancel out" one another at the speed that minimizes the cost of trips of this type.

```
>  plot({cn, cl}, v=50..90, thickness = 3, labels = [speed, cost],
title = `Fig.  3:  cn and cl as function of speed`);
```



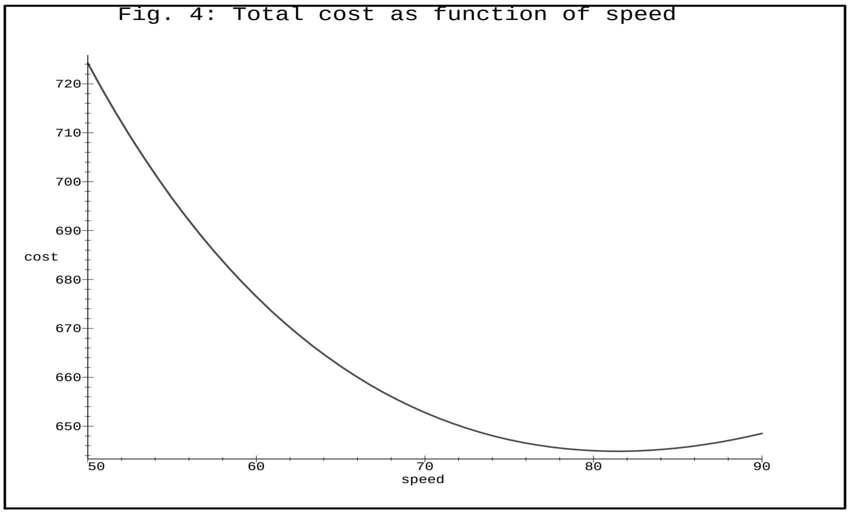
Finally, we obtain the sum of cn and cl, which gives us the total cost:

```
>  ct := cn + cl; #total cost, entire trip
```

$$ct := 200 + \frac{3}{20}\, v^{8/5} + \frac{22290}{v}$$

Plot the cost as a function of speed.

```
>  plot(ct, v=50..90, thickness = 3, labels = [speed, cost], title
= `Fig.  4:  Total cost as function of speed`);
```



These various expressions—the k values and the variable defined for "additional velocity" costs—are the most important structural feature of this simulation. There are many combinations of these values which, although realistic enough, do not have steep slopes or interesting minimization solutions, i.e., conditions that might create incentives for driving dangerously fast. One may find it necessary to run many experiments in order to arrive at a configuration such as the one that initializes this simulation. The major task of empirical investigations, then, would be to ascertain how often dangerous configurations appear in the real world.[note 2]

The function $ct = cn + cl$, simple though it may appear to be, takes on an indefinitely large number of forms depending on the structure of given situations, i.e., the definition of labor components and non-labor components of costs. Because of its high generalizability, it is comparable to Jasso's (1988:12) "axiom of comparison," $J = \ln(\frac{A}{C})$, in which J refers to a justice evaluation, A refers to any sort of actual holding or reward, and C refers to a comparison holding or "just reward" (Jasso, 1996:21). In specific instances A may refer to a combination of factors that influence one's holdings or rewards, and C may refer to a similar set of factors that influence one's expected holdings or rewards [note 3].

There is an infinitude of possible simulations involving the semantics and syntactics of the initializing simulation developed herein. Eventually, empirical tests should be conducted to determine the sorts of structures that actually exist in a given sector of the trucking industry, but the purpose of the present exercise is to develop a large family of logical possibilities that generate a consistent predictive logic comparable to that of Jasso. In the analysis that unfolds below, for instance, we arrive at predictions about the circumstances under which truckers would be tempted to increase speed, or the circumstances under which sanctions against speeding would have a strong prospect of being effective, or the circumstances under which the probability of sanctions would be more important than their severity. The basic underlying model is indeed comparable to Jasso's postulate given above in that it captures the abstract dynamics of the Coleman/Stinchcombe formulation (Coleman, 1968:440; Stinchcombe, 1968:89; Faia, 1986, Ch. 5) involving the interaction of disequilibria and adaptations thereto.

2 Dynamics

In order to analyze the socio-ecological dynamics of the truck-transportation situation as defined thus far, we begin by obtaining a derivative as follows:

```
> Diff(ct, v) = diff(ct, v);
```

$$\frac{\partial}{\partial v}\left(200 + \frac{3}{20}\,v^{8/5} + \frac{22290}{v}\right) = \frac{6}{25}\,v^{3/5} - \frac{22290}{v^2}$$

This expression gives the rate of change of cost, relative to speed. We can use it to find the speed that minimizes total costs; this speed is often very high—over 80 m.p.h. in the initial simulation—and it is sometimes considerably higher.

```
> sol1 := solve(diff(ct, v) = 0, v):  evalf(", 4):
> sol1[1]; evalf(",4);
```

$$92875^{5/13}$$

$$81.42$$

Perhaps more realistically, one could ask how much incentive there would be to increase one's speed from, say, 60 or 70 m.p.h. to any given higher level. That is, how much would costs for each trip drop for each mile-per-hour added to one's speed, starting at a relatively safe speed? Here, we need to solve the derivative for speeds such as $v = 60$ and $v = 70$. Adjacent values may be substituted.

```
> k5 := 60:  k6 := 70:
> subs(v = k5, diff(ct, v)):  evalf(", 3); subs(v = k6, diff(ct, v)):  evalf(", 3);
```

$$-3.38$$

$$-1.48$$

These solutions imply that there is a strong incentive for going faster than 60 m.p.h.—\$3.38 can be shaved from the cost for each additional mile per hour—and that this incentive declines considerably as one approaches what we will soon define as the de facto speed limit. Once familiar with this simulation, users would be able to run experiments by changing various initial constants from *k1* through *k6*, or the variable x_1. Slopes would change, and so would the relative importance of various cost components as developed throughout the simulation.

For the remainder of this paper we shall assume that (given the cost structure as defined heretofore, and as shown in the total-cost function) there is little incentive for increasing speed beyond 85 m.p.h. or so.

3 Adaptation and Social Control

In this example, it appears that the tendency to race for loading docks, as described by Ouellet, could be used by unscrupulous firms as a way of reducing costs: Any factor that causes drivers to increase speed toward the minimal-cost operating speed, which is very high, will reduce costs for the trucking firm. Suppose, however, that these cost-reduction strategies tend to get drivers and firms into trouble with the police and highway patrol. This contingency adds to costs: Firms, we assume, will pay for traffic citations. Estimate these costs on a miles-per-hour basis for all speeds in excess of what we might call the de facto speed limit—the speed where the police start writing tickets. Find out what sort of sanction system is likely to bring about significant speed reductions. For now, there are two decisions about the sanction system: First, what is the probability of a speeding citation on a given trip, when one exceeds the de facto speed limit? Second, what is the severity of the sanction, that is, the cost of speeding citations, for each m.p.h. in excess of the de facto limit?

Social scientists who write about deterrence usually distinguish among four aspects of punishment: probability, severity, celerity (speed of delivery of sanctions), and saliency (awareness of prior warnings or prior instances of punishment). How could all four of these factors be written into a simulation? What is the relative impact of, say, a 10 per cent increase in the probability of punishment versus a plausible increase in the severity of it?

4 Sanctions: I

We begin working toward a resolution of these problems by writing a new cost function, one that will take into account our costs as already established but will also allow for increased costs due to traffic citations. Initially, we assume a fixed probability for traffic citations, regardless of the degree to which speed exceeds the de facto speed limit. The assumption is based on the idea that excessive speeders do all sorts of things to avoid detection; this raises the issue of the relationship between the probability and severity of sanctions.

```
> k7 := 1/10: # probability of receiving a speeding ticket
> k8 := 9: # sanction severity: cost of speeding ticket, for
each m.p.h. excess
> k9 := 75: # the de facto speed limit
> cc := k7*k8*(v - k9); cn; cl;
> ctt := cn + cl + cc; # total cost, including impact of sanction
system
> plot({ct, ctt}, v=k9..90, thickness = 3, labels = [speed, cost],
> title = 'Fig. 5: Excess operating costs due to sanction system,
I');
```

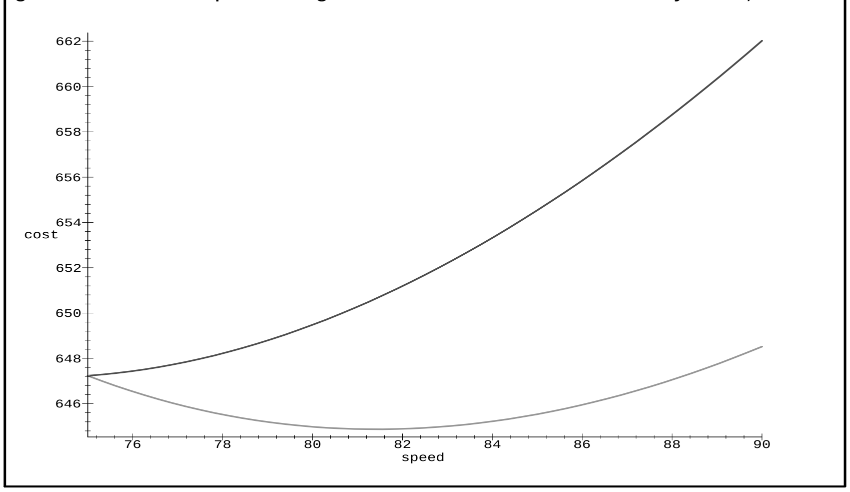
$$cc := \frac{9}{10}\,v - \frac{135}{2}$$

$$200 + \frac{3}{20}\,v^{8/5}$$

$$\frac{22290}{v}$$

$$ctt := \frac{265}{2} + \frac{3}{20}\,v^{8/5} + \frac{22290}{v} + \frac{9}{10}\,v$$

Fig. 5: Excess operating costs due to sanction system, I



How effective is our sanction system? If the region located between these two curves, and ranging from the de facto speed limit to, say, 90 m.p.h., were considered a "braking factor," then we could integrate through the region as a way of assessing the weight of this factor.

```
> Digits := 7: Int(ctt, v = k9..90) = int(ctt, v); i := int(ctt,
v=k9..90): evalf(", 6);
```

$$\int_{75}^{90} \frac{265}{2} + \frac{3}{20} v^{8/5} + \frac{22290}{v} + \frac{9}{10} v \, dv = \frac{265}{2} v + \frac{3}{52} v^{13/5} + 22290 \ln(v) + \frac{9}{20} v^2$$

$$9789.82$$

```
> Int(ct, v = k9..90) = int(ct, v); j := int(ct, v=k9..90): evalf(",
6);
```

$$\int_{75}^{90} 200 + \frac{3}{20} v^{8/5} + \frac{22290}{v} \, dv = 200 v + \frac{3}{52} v^{13/5} + 22290 \ln(v)$$

$$9688.52$$

```
> weight := i - j; evalf(", 5);
```

$$weight := \frac{405}{4}$$

$$101.25$$

This does not seem to be a very heavy weight, but all is relative. How could we make it heavier? What is the relative impact of sanction probability and sanction severity on the braking factor? In a later section, we shall interpret *salience* as a factor that would cause drivers to slow down even in situations where the weight of sanctions has not been increased.

5 Sanctions: II

Although he is not thinking about simulations, Ouellet implies that various ways of organizing the work of truck drivers would have an impact on the dynamics built into the above simulation. In particular, non-union truckers paid on a piece-rate system have strong time pressures, and especially strong incentives for speeding (Ouellet, 1994:29,44). A few companies demand that drivers observe the speed limit (1994:33), and some companies use tachographs to record speeds. But Ouellet presents evidence suggesting that work pressures overcome these rules (1994:44, 47, 48-49, 62, 69) even in the case of drivers paid by the hour:

> Some [drivers] exceeded the speed limit almost constantly, often at gross levels ..., and did so at night when it is difficult to spot police. ... The paradox here is that speeding [under the hourly pay system] had the effect of reducing pay while inviting expensive citations by the police, and accumulating several of these could result in license suspension (Ouellet, 1994:70).

Non-union drivers often do not have the job protections afforded their unionized counterparts; they therefore tend to speed if there are pressures to do so, and they also tend to have accidents and to receive many citations. "In effect [due to poor safety records, etc.], the organization of work in the competitive [non-union] sector tends, over a period of time, to lock the driver into both that sector and its second-rate companies" (Ouellet, 1994:163). Finally, drivers often believe that police harass them for unfair reasons, e.g., working for a small company or being owner-operators themselves.

Ouellet shows that there are many factors that influence a given driver's decisions about speed: the type of company for which she works (including self-employment), the company's policies regarding speed, whether the driver is paid on a piece rate or by the hour, the dynamics of "racing" toward loading docks, the conduct of law enforcement, etc. And there are some interesting paradoxes: For instance, the piece-rate plan is often a method for inducing workers to work faster, whereas an hourly rate tends to slow them down. An hourly rate, however, may give management an incentive for finding unique ways of inducing greater speed on the part of drivers who might otherwise remain reluctant. Many of Ouellet's factors should be included in more complete simulations.

If the police discriminate against certain types of truckers, we should be able to capture this practice mathematically by modifying the following processes, which we have simply copied from above. For instance, one might assume that discrimination occurs in instances where the police arbitrarily lower the de facto speed limit. Suppose it were lowered to 70 m.p.h. What would be the consequences?

```
> k9 := 70:  # the new, discriminatory de facto speed limit
```

```
> cc := k7*k8*(v - k9); cn; cl;
```

```
> ctt := cn + cl + cc; # new total cost, allowing for discrimination
# and including impact of sanction system
```

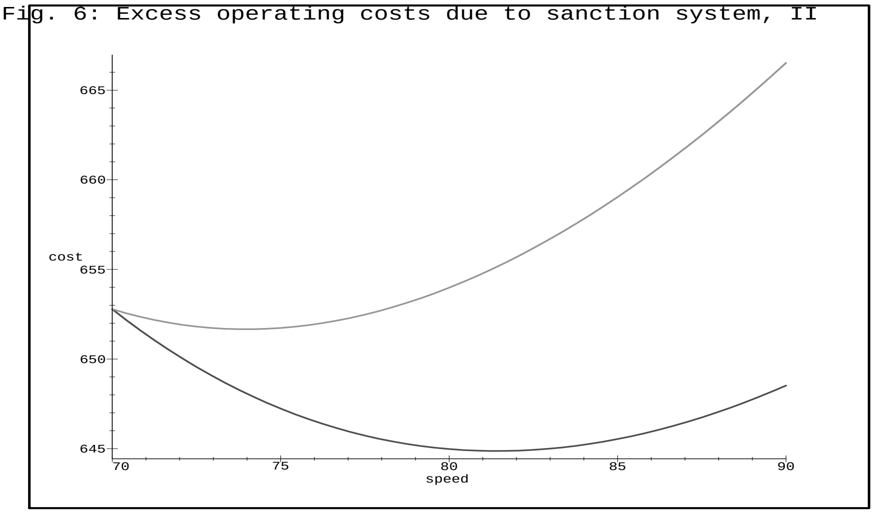
$$cc := \frac{9}{10}\,v - 63$$

$$200 + \frac{3}{20}\,v^{8/5}$$

$$\frac{22290}{v}$$

$$ctt := 137 + \frac{3}{20}\,v^{8/5} + \frac{22290}{v} + \frac{9}{10}\,v$$

```
> plot({ct, ctt}, v=k9..90, thickness = 3, labels = [speed, cost],
title = `Fig. 6: Excess operating costs due to sanction system,
II`);
```




```
> Int(ctt, v = k9..90) = int(ctt, v); i := int(ctt, v=k9..90):
evalf(", 6);
```

```
> Int(ct, v = k9..90) = int(ct, v); j := int(ct, v=k9..90): evalf(",
6);
```

```
> weight := i - j; evalf(", 5);
```

$$\int_{70}^{90} 137 + \frac{3}{20}\, v^{8/5} + \frac{22290}{v} + \frac{9}{10}\, v \, dv = 137\, v + \frac{3}{52}\, v^{13/5} + 22290 \ln(v) + \frac{9}{20}\, v^2$$

$$13117.1$$

$$\int_{70}^{90} 200 + \frac{3}{20}\, v^{8/5} + \frac{22290}{v} \, dv = 200\, v + \frac{3}{52}\, v^{13/5} + 22290 \ln(v)$$

$$12937.1$$

$$\mathit{weight} := 180$$

$$180.$$

Although it is weightier, this sanction system, unlike that depicted in Figure 5, does not create an upward acceleration of costs until speed reaches a level that is well in excess of the de facto speed limit. This may reduce the efficacy of the system, despite the fact that it involves discrimination by law enforcement that may have been intended to bring about substantial reductions of speed by enhancing salience of a relatively low de facto limit. In this instance, then, a discriminatory raising of law-enforcement standards may have a double impact: It may increase the salience of sanctions while at the same time inducing a higher rate of deviance. The next modification of the sanction system appears to overcome this deficiency.

6 Sanctions: III

The expressions *k7* and *k8* should be thought of as scores for variables to which we can attach weights, as in regression analysis. In the equation for the cost of citations, *cc*, the probability and severity of sanctions are multiplied by each other, and then this product is multiplied by the degree to which speed exceeds the de facto limit. In running experiments one might assume, for instance, that for a given amount of funding a law-enforcement agency could raise the severity of sanctions by, say, 15 per cent, or the probability of sanctions by, say, 5 per cent. If *cc* has an impact on the subjective salience of the sanction system among drivers, then one might conclude that raising fines by 15 per cent would have greater deterrent value than raising the probability of citations by 5 per cent. However, *cc* may have little utility as a predictor of subjective awareness of sanctions.

One way to make the sanction system more realistic would be to allow the probability of citations to vary with speed (up to, say, 90 m.p.h.), as in the expression below for x_2. We plot this variable against speed to make sure that it behaves as a probability—ranging, say, from .00 to .7—within the appropriate range for v.

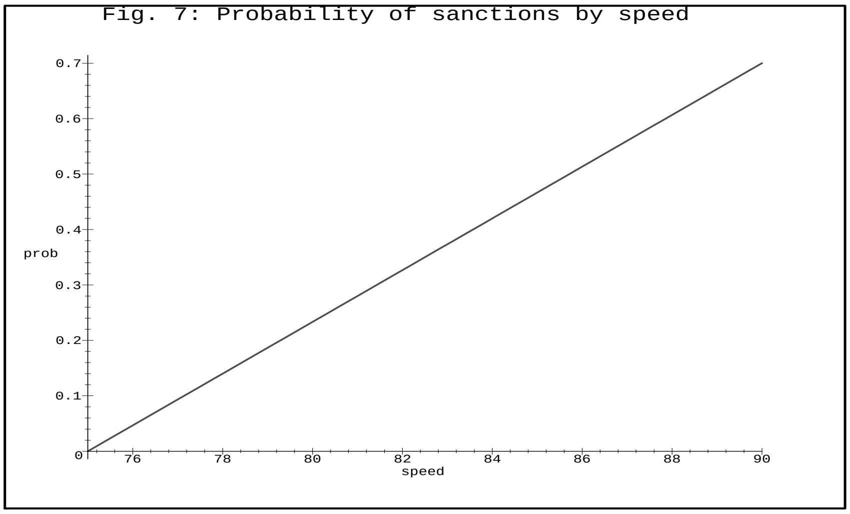
```
> k8 := 9:
```

```
> k9 := 75:  # the de facto speed limit
```

```
> x[2] := (7/10)*(1 - ((90 - v)/(90 - k9))); # probability of receiving
a speeding ticket, for any given trip
```

$$x_2 := -\frac{7}{2} + \frac{7}{150}\,v$$

```
> plot(x[2], v = 75..90, thickness = 3, labels = [speed, prob],
title = `Fig.  7:  Probability of sanctions by speed`);
```



Now, we re-write the crucial equation, substituting x_2 for $k7$:

```
> cc := x[2]*k8*(v - k9); cn; cl;
```

```
> ctt := cn + cl + cc; # total cost, including impact of sanction
system, as modified
```

$$cc := 9\,(-\frac{7}{2} + \frac{7}{150}\,v)\,(v - 75)$$

$$200 + \frac{3}{20}\,v^{8/5}$$

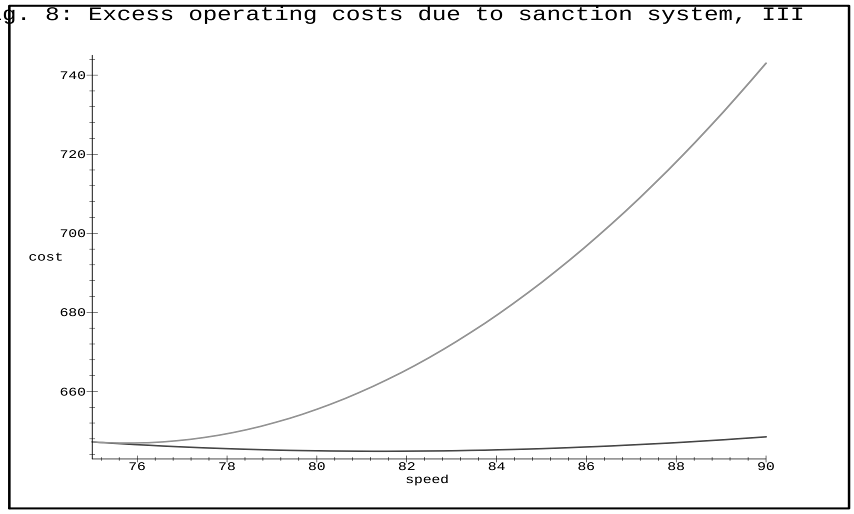
$$\frac{22290}{v}$$

$$ctt := 200 + \frac{3}{20}\,v^{8/5} + \frac{22290}{v} + 9\,(-\frac{7}{2} + \frac{7}{150}\,v)\,(v - 75)$$

Finally, a new plot:

```
> plot({ct, ctt}, v=k9..90, thickness = 3, labels = [speed, cost],
title = `Fig. 8: Excess operating costs due to sanction system,
III`);
```

Fig. 8: Excess operating costs due to sanction system, III




```
> Int(ctt, v = k9..90) = int(ctt, v); i := int(ctt, v=k9..90):
evalf(", 6);
```

```
> Int(ct, v = k9..90) = int(ct, v); j := int(ct, v=k9..90): evalf(",
6);
```

```
> weight := i - j; evalf(", 5);
```

$$\int_{75}^{90} 200 + \frac{3}{20}\,v^{8/5} + \frac{22290}{v} + 9\,(-\frac{7}{2} + \frac{7}{150}\,v)\,(v - 75)\,dv =$$
$$\frac{5125}{2}\,v + \frac{3}{52}\,v^{13/5} + 22290\ln(v) + \frac{7}{50}\,v^3 - \frac{63}{2}\,v^2$$

$$10161.0$$

$$\int_{75}^{90} 200 + \frac{3}{20}\,v^{8/5} + \frac{22290}{v}\,dv = 200\,v + \frac{3}{52}\,v^{13/5} + 22290\ln(v)$$

$$9688.52$$

$$weight := \frac{945}{2}$$

$$472.50$$

7 Sanctions: IV

If the preceding equation for *cc* captures the objective (and perhaps subjective) dimensions of the sanction system as it impinges on speeding, then the values obtained for sanction weight might lead us to predict that, contrary to what we often observe regarding the relative importance of sanction probability and severity, in this instance severity is likely to have the larger impact. This is due to the fact that, other things equal, it may be much easier to make objective changes in sanction severity than in sanction probability. If violators of traffic laws tend to play highly rational games with law enforcement (Chambliss, 1966; Gibbs, 1975; Nettler, 1976), it is possible that such violators would be far more responsive to feasible changes of sanction severity than to feasible changes of sanction probability.

Another factor that influences the relative impact of sanction probability and severity is that, in many instances, an increase in severity brings about a decrease in probability, because the judicial system is reluctant to impose weightier sanctions. It is as if there were a negative correlation between severity and probability, such that, for instance, the following regression equation would be appropriate:

```
> prbmult := 1 - (3/10)*(sevmult - 1); sevmult := 2.2; # adjust
the immediately preceding expression, using sevmult = 2.2 for maximum
sanction efficacy
```

$$prbmult := \frac{13}{10} - \frac{3}{10}\,sevmult$$

$$sevmult := 2.2$$

```
> prbmult;
```

$$.6400000$$

It would then be feasible to experiment with variations in probability and severity of sanctions, as follows. First, we collect some earlier terms:

```
> k8 := 9:  # sanction severity:  cost of speeding ticket, for
each m.p.h.  excess
```

```
> k9 := 75:  # the de facto speed limit
```

```
> x[2] := (7/10)*(1 - ((90 - v)/(90 - k9))); # probability of receiving
a speeding citation, for any given trip
```

Finally, we write a new equation incorporating the correlation between probability and severity of sanctions [note 4]:

```
> cc := prbmult*x[2]*sevmult*k8*(v - k9); cn; cl;
```

```
> ctt := cn + cl + cc; # total cost, including impact of sanction
system
```

$$x_2 := -\frac{7}{2} + \frac{7}{150}\,v$$

$$cc := 12.67200\,(-\frac{7}{2} + \frac{7}{150}\,v)\,(v - 75)$$

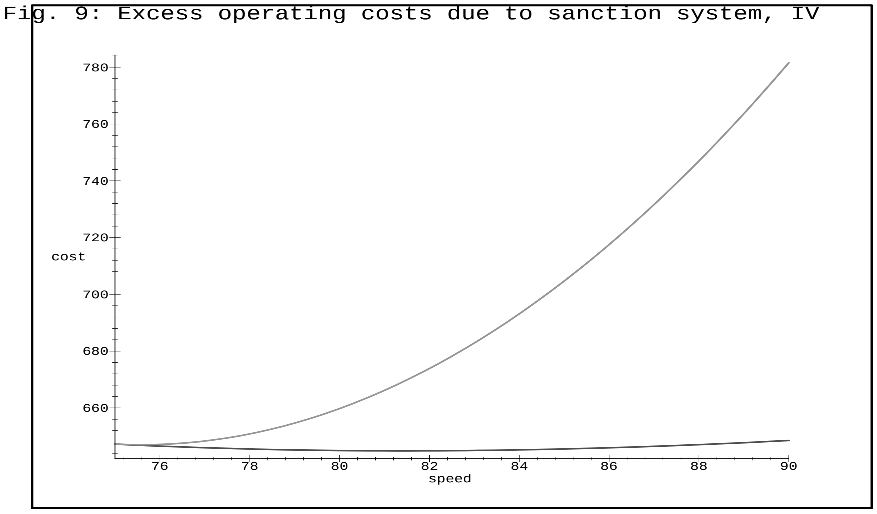
$$200 + \frac{3}{20}\,v^{8/5}$$

$$\frac{22290}{v}$$

$$ctt := 200 + \frac{3}{20}\,v^{8/5} + \frac{22290}{v} + 12.67200\,(-\frac{7}{2} + \frac{7}{150}\,v)\,(v - 75)$$

And a new plot, followed by the calculation for sanction weight:

```
>  plot({ct, ctt}, v=k9..90, thickness = 3, labels = [speed, cost],
title = `Fig.  9:  Excess operating costs due to sanction system,
IV`);
```




```
>  Int(ctt, v = k9..90) = int(ctt, v); i := int(ctt, v=k9..90):
evalf(", 6);
```

```
>  Int(ct, v = k9..90) = int(ct, v); j := int(ct, v=k9..90):  evalf(",
6);
```

```
>  weight := i - j; evalf(", 5);
```

$$\int_{75}^{90} 200 + \frac{3}{20}\,v^{8/5} + \frac{22290}{v} + 12.67200\,(-\frac{7}{2} + \frac{7}{150}\,v)\,(v - 75)\,dv =$$

$$3526.400\,v + .05769231\,v^{13/5} + 22290.\ln(v) + .1971200\,v^3 - 44.35200\,v^2$$

$$10353.9$$

$$\int_{75}^{90} 200 + \frac{3}{20}\, v^{8/5} + \frac{22290}{v}\, dv = 200\, v + \frac{3}{52}\, v^{13/5} + 22290 \ln(v)$$

$$9688.52$$

$$weight := 7353.94 - 22290 \ln(2) - 22290 \ln(3) + 22290 \ln(5) - \frac{6075}{13}\, 90^{3/5} + \frac{16875}{52}\, 75^{3/5}$$

$$665.2$$

Experiments show that, when the multiplier for sanction severity is rotated from .2 through 1, sanction weight increases from 117 to 472 (as in the preceding section). Then, as sanction severity is raised still further, sanction weight increases to a maximum that is around 665 when the severity multiplier is set at 2.2 (and the probability multiplier becomes .64), but then it declines to a value of 374 when sanction severity is multiplied by 3.6 (and the probability drops accordingly). The strategy of increasing the severity of sanctions, then, eventually may become counterproductive. Examining the same pattern from right to left, we would note that an increasing probability of sanctions would raise the overall weight of sanctions (Sherman and Berk, 1984), but after this probability had moved through the zone of maximum efficacy we would again observe a decline in efficacy. Thus, one might conclude that if there is a u-shaped curve for the size of prison populations relative to severity of prison sentences (Clark and Lee, 1996), then there is probably also such a curve, with an inverted u shape, for the impact of sanctions relative to their severity.

8 Salutary Checks and Processor Time

It is a good idea to check complicated integrals and other mathematical paraphernalia. For *ct* and *ctt*, we shall restate the function, obtain its integral symbolically, obtain its integral numerically, and then obtain the derivative of the integral. This procedure should bring us back to the original function. Due to the complexity of *ctt*, I request an additional algebraic manipulation, as shown.

```
> ct; Int(ct, v) = int(ct, v); Diff(", v) = diff(", v);
```

$$200 + \frac{3}{20}\, v^{8/5} + \frac{22290}{v}$$

$$\int 200 + \frac{3}{20}\, v^{8/5} + \frac{22290}{v}\, dv = 200\, v + \frac{3}{52}\, v^{13/5} + 22290 \ln(v)$$

$$\frac{\partial}{\partial v}\left(\int 200 + \frac{3}{20}\, v^{8/5} + \frac{22290}{v}\, dv = 200\, v + \frac{3}{52}\, v^{13/5} + 22290 \ln(v)\right) = \\ (200 + \frac{3}{20}\, v^{8/5} + \frac{22290}{v} = 200 + \frac{3}{20}\, v^{8/5} + \frac{22290}{v})$$

```
> ctt;
```

$$200+\frac{3}{20}v^{8/5}+\frac{22290}{v}+12.67200\,(-\frac{7}{2}+\frac{7}{150}v)\,(v-75)$$

Here we obtain a simplified version of *ctt*.

```
> expand(");
```

$$3526.400+\frac{3}{20}v^{8/5}+\frac{22290}{v}-88.70400\,v+.5913600\,v^2$$

```
> Int(ctt, v) = int(ctt, v);
```

$$\int 200+\frac{3}{20}v^{8/5}+\frac{22290}{v}+12.67200\,(-\frac{7}{2}+\frac{7}{150}v)\,(v-75)\,dv =$$
$$3526.400\,v+.05769231\,v^{13/5}+22290.\ln(v)+.1971200\,v^3-44.35200\,v^2$$

Notice that, following the next command, the final output line should return the original function, *ctt*. First, we define an expression that refers to the derivative of the above integral:

```
> expr1 := Diff(", v) = diff(", v);
```

$$\mathit{expr1} := \frac{\partial}{\partial v}\left(\int 200+\frac{3}{20}v^{8/5}+\frac{22290}{v}+12.67200\,(-\frac{7}{2}+\frac{7}{150}v)\,(v-75)\,dv =\right.$$
$$\left. 3526.400\,v+.05769231\,v^{13/5}+22290.\ln(v)+.1971200\,v^3-44.35200\,v^2\right) = ($$
$$200+\frac{3}{20}v^{8/5}+\frac{22290}{v}+12.67200\,(-\frac{7}{2}+\frac{7}{150}v)\,(v-75) =$$
$$3526.400+.1500000\,v^{8/5}+\frac{22290.}{v}+.5913600\,v^2-88.70400\,v)$$

Now, we isolate the right-hand side of this elaborate expression:

```
> rhs(expr1);
```

$$200+\frac{3}{20}v^{8/5}+\frac{22290}{v}+12.67200\,(-\frac{7}{2}+\frac{7}{150}v)\,(v-75) =$$
$$3526.400+.1500000\,v^{8/5}+\frac{22290.}{v}+.5913600\,v^2-88.70400\,v$$

The left-hand side of the above expression appears to be *ctt*. Let's check, by first isolating the left-hand side and then by obtaining a boolean test of the appropriate equation:

```
> expr2 := lhs(");
```

$$\mathit{expr2} := 200+\frac{3}{20}v^{8/5}+\frac{22290}{v}+12.67200\,(-\frac{7}{2}+\frac{7}{150}v)\,(v-75)$$

```
> ctt;
```

$$200+\frac{3}{20}v^{8/5}+\frac{22290}{v}+12.67200\left(-\frac{7}{2}+\frac{7}{150}v\right)(v-75)$$

On the basis of inspection things appear to be reassuring, but in order to make sure we provide a boolean test:

```
> evalb(expr2 = ctt);
```

$$true$$

Q.E.D.

The final expression, for job time, tells us that the typical experiment using this program (up to this point) is likely to require over 20 seconds of computer processor activity. The variable called *job*, defined at the top of the program, copies one's computer clock time between the beginning and the end of an experiment; the next command merely subtracts the beginning clock time from the ending clock time. Repeat this command below, following further experiments as necessary.

```
> time() - job;
```

$$25.964$$

Notes

(1) This paper approaches such questions by means of a complete Maple V (release 4.00a) program. Excellent documentation is distributed with the package itself, and additional documentation appears at:

www.mathematics.com/Products/MapleV/MapleV.html

Users can execute the program interactively, one step at a time, or as a batch process. Each Maple input command is preceded by the "greater than" (>) symbol, although this symbol may be system dependent.

(2) Transportation technology develops rapidly, and it is entirely possible that the global positioning system (GPS) will add substantial flexibility to the structural conditions defined in this paper. To cite one possibility, the GPS may give dispatchers the ability to suggest speed changes at frequent intervals. It is this sort of rapid adjustment—concurrent feedback with high frequency of change and perhaps low amplitude—that enables a few airline pilots to make transcontinental flights in which they save several thousands of dollars in fuel.

For an illustration of the way in which Joe DiMaggio's record for consecutive games with at least one base hit may have been influenced by concurrent feedback, see Lieberson (1997:15-16).

(3) In one of her applications (1996), for example, Jasso shows how a relatively simple justice evaluation would be calculated for a migrant. In the equations below, the term z_d refers to a migrant's sense of well-being (or reward) at a given place of destination, and z_o is the same value for the place of origin. The expression cz, referring to a change of z or well-being, is a before-after comparison in which each term, z_d and z_o, can be written in the logarithmic form of the axiom of comparison.

The following Maple input lines tell us that, at a place of origin, a potential migrant evaluates her situation by comparing her own actual reward (e.g., wealth), called x_o, against the average wealth of the place of origin, which is the "just reward." In the Maple input this average, a mean, is written into the denominator as $\frac{s_o}{n_o}$, or total wealth at the place of origin divided by the population at the place of origin. Maple inverts this divisor and multiplies.

```
> z[o] := ln(x[o]/(s[o]/n[o]));
```

$$z_o := \ln(\frac{x_o\, n_o}{s_o})$$

We then carry out the same manipulations for the place of destination.

```
> z[d] := ln(x[d]/(s[d]/n[d]));
```

$$z_d := \ln(\frac{x_d\, n_d}{s_d})$$

Next we obtain the difference, i.e., the change in the migrant's sense of well-being (or in her justice evaluation) brought about by having migrated:

```
> cz := z[d] - z[o];
```

$$cz := \ln(\frac{x_d\, n_d}{s_d}) - \ln(\frac{x_o\, n_o}{s_o})$$

Many of Jasso's "predictions" are derived by examining the partial derivatives of relatively complex formulations of this sort of function, with a view toward discovering changes in the sense of well-being brought about by changes of one (or several) of the variables. For instance, Jasso's migration model eventually arrives at the prediction that, *ceteris paribus*, if the average wealth at the place of destination were raised, migration would create a reduction of the sense of well-being due to the greater wealth surrounding a newly arrived in-migrant.

For another illustration of the utility of abstract formulas in generating specific hypotheses, see Weeks' (1996:487-88) discussion of environmental-impact research.

(4) The regression line that corresponds to this correlation has the following properties: (a) it passes through the point *prbmult* = 1 and *sevmult* = 1; (b) with *prbmult* defining the ordinate, the line has a slight downward slope reflecting the assumption that *sevmult* has greater elasticity than *prbmult*; (c) usable scores for *sevmult* start at about .2, and it seems plausible that their highest value would impose a value of .2 on *prbmult*, with results as follows:

```
> prbmult := 'prbmult'; sevmult := 'sevmult'; # Re-initialize variables
```

$$prbmult := prbmult$$

$$sevmult := sevmult$$

```
> prbmult := 1 - (3/10)*(sevmult - 1);
```

$$prbmult := \frac{13}{10} - \frac{3}{10}\, sevmult$$

```
> subs(sevmult = 2, prbmult); # Substitute value
```

$$\frac{7}{10}$$

```
> solve({prbmult = 1 - (3/10)*(sevmult - 1), prbmult = 2/10}, sevmult);
# Substitute value;
```

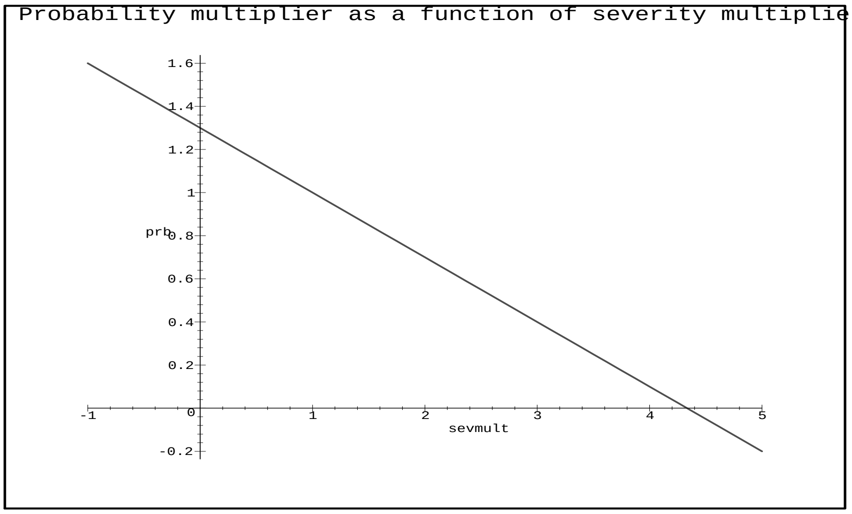
$$\{sevmult = \frac{11}{3}\}$$

```
> evalf(", 3);
```

$$\{sevmult = 3.67\}$$

```
> plot(prbmult, sevmult=-1..5, thickness=3, title = `Fig. 10:
Probability multiplier as a function of severity multiplier`, labels
= [sevmult, prb]);
```

Fig. 10: Probability multiplier as a function of severity multiplier



In our own experiments, *sevmult* ranges from .2 to 3.6.

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