Mathematica and didactical innovation: a quadric use case

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Abstract. A package for the education in mathematics focuses on the study of a quadric is presented. The package is structured in an hypertextual way and use a "visual" (animations and graphs) approach to mathematical concepts which seems the most appropriate from both the pedagocical and experimental point of view. It includes a theoretical part and a guided lab for the practical session. The most meaningful innovation is given by the verification routines, so that the package gives an essentially self-contained, complete environment. The user may solve exercises by hand, check the solutions he/she finds using two verification functions and receive suggestions.

1. Introduction

Since the first time they were introduced, Mathematica and all similar C.A.S.(Computer Algebra Systems), have been useful to change both the way people looked and thought of didactics in mathematics. There is particular attention to a new way of teaching and learning: this is due to the new technologies (multimedia and web) which are a common instrument and to the necessity of gearing to european standards.

Students cannot be no longer passive watchers of a process which used to be distant and unrelated to them. The computer is part of their life and it is a a tool that bags the attention of the young people, so traditional classes are not enough for them! The school needs of those communicative tools that our time offers, otherwise progress is not possible. This is the main reason of new school/university experimental programs.

Last year the Faculty of Engineering of the University of Salerno experimented a new type of class, using multimedia packages: the faculty offered classical (blackboard and chalk) lessons and at the same time laboratory lessons where the students were introduced to the study of the same subjects by using packages developed in Mathematica[TM]. These packages has been jointly realised at the Department of Computer Science and Applied Mathematics at the Faculty of Engineering and the CRMPA -Centre for Research in Pure and Applied Mathematics - situated in the University of Salerno.

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The aim of this work is to describe one of the packages for the education in mathematics focused on the study of a quadric. The package is structured in an hypertextual way, including a theoretical part of and a guided lab for the practical session. In the tutorial the quadrics are introduced starting from the definition and proceeding with the classification and the reduction to a canonical form using the rotation/translation method.

The laboratory part has been given a structure designed in order to make the final user's interface as friendly as possible. Such an improvement has been obtained by the insertion of buttons and links that make it possible to use all implemented functions without prior knowledge of their syntax. The latter functions allow the user to choose among randomised exercises, whose step-by-step automatic solution (classification, canonical form reduction and graph) may be visualised. The most meaningful innovation is, nonetheless, given by the verification routines. The user may solve exercises by hand and check the solutions he/she finds using two verification functions. The first is entirely automatic: the user enters, as a string, the class (degenerate or not), type(e.g. paraboloid) and shape (with or without centre) of the quadric. The routine detects mistakes (using logical inferences), and suggests possible theoretical deepening. Moreover, a manual verification routine has been implemented, allowing to visually verify the values of the parameters that characterise the quadric. Again, for the sake of completeness, a subsidiary routine has been implemented that takes as an input the polynomial associated with the quadric, thus allowing the use of the previously described functions.

We stress the fact that the package gives an essentially self-contained, complete environment. In fact, every fundamental step in the study of the chosen subject is covered, from theory to various kinds of exercises. In particular, some diagnostic functionalities are implemented that allow the student to make exercises, have them automatically corrected and even receive suggestions on how to recover from errors.

2. State of the art

Analysing the schedules of some projects published on conference proceedings, we can find out that at the moment different strategies of implementation of mathematical/scientific teaching packages are available. We cite just some examples (many others may be found):

- TRANSMATH is developing mathematics courseware based on Toolbook^{TM} (hypertext) documents for Microsoft Windows^{TM}, using Mathematica^{TM} as an intelligent support engine for mathematical activities [7];

- “Transitional Mathematics Project” [4] [5], developed by the Imperial College in London, which basically consists in a pre-course on the use of Mathematics and than a course on a few mathematics algorithm;

- the project “Rethinking the way we teach undergraduate physics and engineering with Mathematica” [3];

- MEDIT - Multimedia Environment for Distributed Interactive Teaching - developed at theSwiss Federal Institute of Technology [1].

Other projects basically utilise the potentiality of the engine of Mathematica for symbolic numeric and graphic computations, sending the results to the main environment as icons. An example of this strategy is Hypermedia Mathematical Learning Environment (HMLE) [2], developed by the Department of Mathematics of Tampere University of Technology, which consist on an interface developed in C/C++ connected to Mathematica via MathLink. Another example of the same strategy is the development of a prototype by the University of Leeds through the use of Toolbook to simplify the user interface.

3. The package: the main features

The package on the quadrics has been used for supporting the learning process of …rst year Engineering students’ on this subject. Lectures have been given introducing the theoretical concepts: what a quadric is, quadrics’ classi…cation, the standard form of a quadric, how to classify and reduce to standard form, and some examples. Then the students have been split into small groups (20 students for each group) and a one-hour exercise session was held weekly in a computer classroom. During the computer classroom the students have used the package on the study of a quadric.

The package can be split into two parts: the theoretical one and the practical one. Note that each of them is “dynamical” in the following sense. The theoretical concepts are introduced by using a Mathematica notebook: the material included in this notebook is the same that teachers can use during a classical classroom but it is presented in an interactive way. Thus for example quadrics were typically introduced starting from those quadrics which are generated by a rotation of a conic. It is not possible to “show” this to students with chalk-and-blackboard: they can just “imagine” the rotating conic! But by means of animation tools of Mathematica™, this is possible! In fact, in the theory notebook the student can e¤ectively view how a quadric arises from a rotation of a conic: he only has to click on a …gure and an animation starts and there the quadric is! Moreover, in the case of the hyperbola, he/she can see that the generated quadric is di¤erent as the rotation axis changes. It is well estabilished, from the pedagogical point of view, that the impact of "visible" concepts is more efficient than any other: imagines stimulate the intuition and help deep understanding and memorization.

According to this approach, two other facilities are offered in order to "see" the quadric: the student can ask for the standard graph (which means "canonical" graph) or for the "exact" graph (which means the graph in the original coordinate system); he/she can choose to see the graph of the quadric before starting the computation, so that he can recognise it just by looking at the figure, or he/she can check graphically if his/her result is correct. If the quadric has no real part, a message is given. The graphs are obtained using the plotting functions of Mathematica™.

The students have two alternatives for choosing an exercise: a) choose a randomly generated quadric; b) give a quadric he/she took from a book or he/she invented. In both cases they can choose between a verification function or an automatic solution.

The verification routine has been very useful for students. In fact they can verify their level of learning: the student has to give the routine the classification of the given quadric, i.e. the class (degenerate or not), the type (paraboloid, hyperboloid of one or two sheets, ellipsoid or some couple of planes) and the shape (central or not); if some of the terms he/she gives is not correct, the verification routine points it out and explains which step of the student's process for classification is most probably wrong, so the student can study thoroughly the related theory as presented in the package. The routine is actually able to do slightly more than this: in fact, it is able to recognise errors of a (most probably) theoretical character (e.g. logical inconsistencies) and computational errors. Correspondingly, a different warning message is generated, suggesting the most likely nature of the error and suitable means of correcting it (within the package's environment). This feature proved particularly useful in saving time during the error correction phase, since students did not have to unduely repeat the whole theoretical background in case of mere computational errors and, conversely, received a timely warning when they needed to get a better understanding of the underlying theory. Therefore, the previous functionality can be regarded - at least from the experimental points of view - as a kind of support, increasing the success rate for a final examination on the subject.

The canonical form has been implemented by using the rotation-translation method. The student can check each step he/she has to take in order to have the canonical form: in fact the package provides the equations of the rotation and of the translation and it explains how to get those equations. We want to spend few words to give more details: for each rotation there is a theoretical recall on how to get the rotation, then computations are made for the specific case, which means the eigenvalues, the eigenvectors and the rotation matrix are constructed, then the rotation is applied to the input quadric and a new equation is given; for each translation the exact needed steps are explained, this means that there are di¤erent output and theoretical recalls according to the speci…c given exercise (e.g. square completion, etc.). We want to underline thefollowing point: in many textbook something like "as we saw for the conics, there are suitable rotations and translations in order to have the canonical form": this is not really true! An example is given by the parabolic cylinder: if the students apply the classical steps for the rotation and translation, he/she does not get the canonical form, but they need to apply another rotation and another translation. From our experience this is a critical point for the students, if they do not have a guide to overcome this di¢culty, whilst the use of the presented package can help them so that they can save time.

4. Examples

We explore some examples of exercise modules implemented with Mathematica 3.0.

First of all, the package o¤ers to the students the possibility to classify a given quadric or a randomly generated one. Students can also check the solving procedure step by step.

The classification of the quadrics

Start

Exercise palettes Help

Give a quadric | Random quadric | Help | Example | Teory

Standard graphs

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Classify the following qudric:

$$5 - \frac{1004\,x^2}{321} - \frac{6752\,x\,y}{321} + \frac{4276\,y^2}{321} + 4\,z + \frac{10468\,x\,z}{321} - \frac{5188\,y\,z}{321} + \frac{8926\,z^2}{321} = 0$$

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Classification | Canonical form | Verification

The canonical form is:

$$\frac{4815\left(\frac{23464}{4815} + 40\,X^2 - 12\,Y^2 + 10\,Z^2\right)}{23464} = 0$$

Options | New exercise

Classification | Canonical form | Verification

Rotation | Translation

Graph | Standard Graph

The quadratic matrix is:

$$A = \begin{pmatrix} -\frac{1004}{321} & -\frac{3376}{321} & \frac{5234}{321} \\ -\frac{3376}{321} & \frac{4276}{321} & -\frac{2594}{321} \\ \frac{5234}{321} & -\frac{2594}{321} & \frac{8926}{321} \end{pmatrix}$$

The matrix of the needed rotation is made as following:

its columns are an orthonormal base of R^3

whose vectors are eigenvectors of A.

So you need to compute the eigenvalues of A

and an orthonormal base of each eigenspace

The eigenvalues of A (ordered such that $|a|>|b|$) are :

$\{40, -12, 10\}$

The corresponding orthonormal eigenvectors are :

$$\left\{\left\{\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \sqrt{\frac{2}{3}}\right\}, \left\{-\frac{23}{\sqrt{642}}, -\frac{7}{\sqrt{642}}, 4\sqrt{\frac{2}{321}}\right\}, \left\{-\frac{1}{\sqrt{107}}, \frac{9}{\sqrt{107}}, \frac{5}{\sqrt{107}}\right\}\right\}$$

Note that the determinant of the matrix of the eigenvectors is -1

so you need to substitute one of the vectors with its opposite.

Then the rotation matrix is

$$\begin{pmatrix} -\frac{1}{\sqrt{6}} & -\frac{23}{\sqrt{642}} & -\frac{1}{\sqrt{107}} \\ \frac{1}{\sqrt{6}} & -\frac{7}{\sqrt{642}} & \frac{9}{\sqrt{107}} \\ -\sqrt{\frac{2}{3}} & 4\sqrt{\frac{2}{321}} & \frac{5}{\sqrt{107}} \end{pmatrix}$$

The equations of the rotation are:

$$x = \frac{-X}{\sqrt{6}} - \frac{23\,Y}{\sqrt{642}} - \frac{Z}{\sqrt{107}}$$

$$y = \frac{X}{\sqrt{6}} - \frac{7\,Y}{\sqrt{642}} + \frac{9\,Z}{\sqrt{107}}$$

$$z = -\sqrt{\frac{2}{3}}\,X + 4\sqrt{\frac{2}{321}}\,Y + \frac{5\,Z}{\sqrt{107}}$$

The new equation of the quadric:

$$5 - 4\sqrt{\frac{2}{3}}\,X + 40\,X^2 + 16\sqrt{\frac{2}{321}}\,Y - 12\,Y^2 + \frac{20\,Z}{\sqrt{107}} + 10\,Z^2 = 0$$

Options **New exercise**

Classification **Canonical form** **Verification**

Rotation **Translation**

Graph **Standard Graph**

In order to obtain the translation equation you need

to complete the square in X, Y, Z

Note that:

$$a\ x^2 + b\ x = a\ \mathrm{H}x^2 + \frac{b}{a}\ x\mathrm{L} = a\ @\ \mathrm{J}x + \frac{b}{2\,a}\mathrm{N}^2 - \frac{b^2}{4\,a^2}\ \mathrm{D}$$

So you have:

$$X = \frac{1}{10\sqrt{6}} + X$$

$$Y = \frac{2\sqrt{\frac{2}{321}}}{3} + Y$$

$$Z = \frac{-1}{\sqrt{107}} + Z$$

The new equation of the quadric is :

$$\frac{23464}{4815} + 40\,X^2 - 12\,Y^2 + 10\,Z^2$$

The new equation of the quadric is :

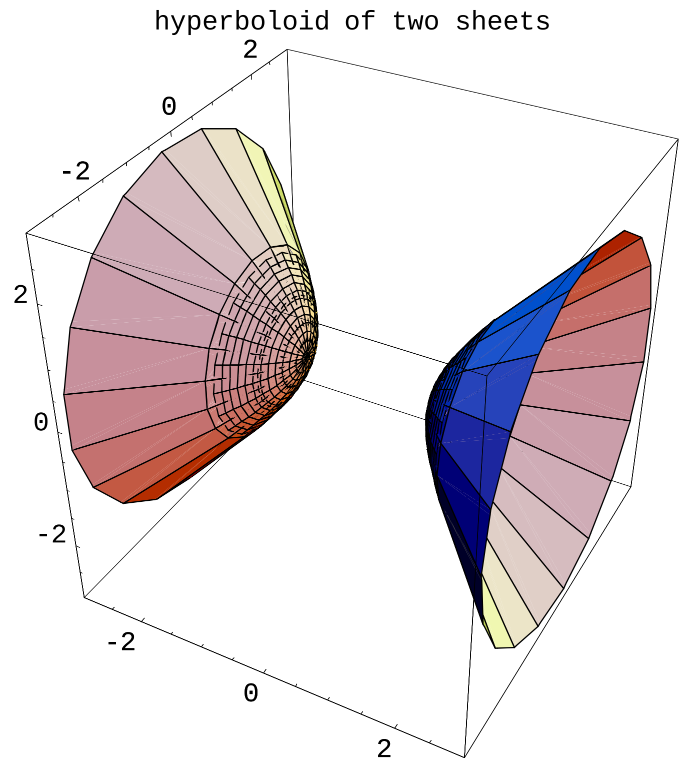
$$\frac{23464}{4815} + 40\,X^2 - 12\,Y^2 + 10\,Z^2$$

Options **New exercise**

Classification **Canonical form** **Verification**

Rotation **Translation**

Graph **Standard Graph**



Options New exercise

Classification Canonical form Verification

Rotation Translation

Graph Standard Graph

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Verification@"degenerate", "sphere", "non central"D
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The classification is quite wrong.

It might be better to have another look at the theory.

Anyway a sphere is central

The given quadric is degenerate, so the computation

of the determinant of the quadric matrix is wrong.

The given quadric is not a sphere and it is not non central

The determinant of the quadratic matrix is not zero.

Options | New exercise

Give a quadric | Random quadric | Help | Example | Teory | Standard graphs

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GiveAQuadric@2 x y - x - y + zD
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Classification | Canonical form | Verification

The quadric matrix is:

$$A1 = \begin{pmatrix} 0 & 1 & 0 & -\frac{1}{2} \\ 1 & 0 & 0 & -\frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$$

and its determinant is $\frac{1}{4}$

The quadratic matrix is :

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

and its determinant is 0

The eigenvalues of A are: -1,0,1

The first step is to estabilish if the quadric is degenerate or not

checking if the determinant of the quadric matrix is zero or not.

Since in this case it is not zero, the quadric is non degenerate.

In particular the determinant is greater than zero.

Now you need to check if the determinant of the quadratic matrix is zero or not.

In this case it is zero, so the quadric is not central

The quadric is a hyperbolic paraboloid.

Options New exercise

Classification Canonical form Verification

Rotation Translation

Graph Standard Graph

The canonical form is:

$X^2 - Y^2 + Z = 0$

Options New exercise

Classification Canonical form Verification

Rotation Translation

Graph Standard Graph

The quadratic matrix is:

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

The matrix of the needed rotation is made as following:

its columns are an orthonormal base of R^3

whose vectors are eigenvectors of A.

So you need to compute the eigenvalues of A

and an orthonormal base of each eigenspace

The eigenvalues of A Hordered such that ÈaÈ>ÈbÈL are :

81, -1, 0<

The corresponding orthonormal eigenvectors are :

$99\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0=, 9-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0=, 80, 0, 1<=$

Then the rotation matrix is

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The equations of the rotation are:

$x = \frac{X}{\sqrt{2}} - \frac{Y}{\sqrt{2}}$

$y = \frac{X}{\sqrt{2}} + \frac{Y}{\sqrt{2}}$

$z = Z$

The new equation of the quadric:

$-\sqrt{2}\, X + X^2 - Y^2 + Z = 0$

Options New exercise

Classification Canonical form Verification

Rotation Translation

Graph Standard Graph

In order to obtain the translation equation you need

to complete the square in X, Y

Note that:

$a\ x^2 + b\ x = a\ Hx^2 + \frac{b}{a}\ xL = a\ @\ Jx + \frac{b}{2\ a}N^2 - \frac{b^2}{4\ a^2}\ D$

So you have:

$X = \frac{1}{\sqrt{2}} + X$

$Y = Y$

Moreover you need to eliminate the constant term.

In order to do this, note that

$c z + d = c Jz + \frac{d}{c} N$

So $Z = \frac{1}{2} + Z$

The new equation of the quadric is :

$X^2 - Y^2 + Z$

The new equation of the quadric is :

$X^2 - Y^2 + Z$

Options | New exercise

Classification | Canonical form | Verification

Rotation | Translation

Graph | Standard Graph

Give a quadric | Random quadric | Help | Example | Teory | Standard graphs

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GiveAQuadric@x^2 + y^2 + z^2 + 2 x y - 2 x z - 2 y z - 2 x + 4 y - 4 z - 3D
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Classification | Canonical form | Verification

The quadric matrix is:

$$A1 = \begin{pmatrix} 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & 2 \\ -1 & -1 & 1 & -2 \\ -1 & 2 & -2 & -3 \end{pmatrix}$$

and its determinant is 0

The quadratic matrix is :

$$A = \begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix}$$

and its determinant is 0

The eigenvalues of A are: 0,0,3

The first step is to check if the quadric is degenerate or not

checking if the determinant of the quadric matrix is zero or not.

In this case it is zero, so the quadric is degenerate.

Now you need to compute the rank of the matrices A1 and A, and you have

rank of A = 1

rank of A1 = 3

Since the determinant of A is zero, the quadric is not central.

Since the rank of A1 is 3, you need to check the sign of the eigenvalues

Since two eigenvalues are zero, the quadric is a parabolic cylinder

Options New exercise

Classification Canonical form Verification

Rotation Translation

Graph Standard Graph

The canonical form is:

$3X^2 + 2\sqrt{6}\,Y = 0$

Options New exercise

Classification Canonical form Verification

Rotation Translation

Graph Standard Graph

The quadratic matrix is:

$$A = \begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix}$$

The matrix of the needed rotation is made as following:

its columns are an orthonormal base of R^3 whose vectors are eigenvectors of A.

So you need to compute the eigenvalues of A

and an orthonormal base of each eigenspace

The eigenvalues of A (ordered such that $|a| \geq |b|$) are :

$\{3, 0, 0\}$

The corresponding orthonormal eigenvectors are :

$$\left\{\left\{-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right\}, \left\{-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right\}, \left\{\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \sqrt{\frac{2}{3}}\right\}\right\}$$

Note that the determinant of the matrix of the eigenvectors is -1

so you need to substitute one of the vectors with its opposite.

Then the rotation matrix is

$$\begin{pmatrix} \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ -\frac{1}{\sqrt{3}} & 0 & \sqrt{\frac{2}{3}} \end{pmatrix}$$

The equations of the rotation are:

$$x = \frac{X}{\sqrt{3}} - \frac{Y}{\sqrt{2}} + \frac{Z}{\sqrt{6}}$$

$$y = \frac{X}{\sqrt{3}} + \frac{Y}{\sqrt{2}} + \frac{Z}{\sqrt{6}}$$

$$z = -\frac{X}{\sqrt{3}} + \sqrt{\frac{2}{3}}\, Z$$

The new equation of the quadric:

$$-3 + 2\sqrt{3}\, X + 3X^2 + 3\sqrt{2}\, Y - \sqrt{6}\, Z = 0$$

Options | New exercise

Classification | Canonical form | Verification

Rotation | Translation

Graph | Standard Graph

In this case you need to do a first translation

in order to eliminate the linear term in X so you have to complete the square in X

Note that:

$$a\ x^2 + b\ x = a\ Hx^2 + \frac{b}{a}\ xL = a\ @\ Jx + \frac{b}{2\ a}N^2 - \frac{b^2}{4\ a^2}\ D$$

So you have:

$$X = -\frac{1}{\sqrt{3}} + X$$

$$Y = Y$$

$$Z = Z$$

The new equation of the quadric is :

$$-4 + 3\,X^2 + 3\sqrt{2}\ Y - \sqrt{6}\ Z$$

Now you have to do a new rotation in order to eliminate the linear term in Z

Note that if you have $a\,x^2 + b\,y + c\,z + d = 0$,

the rotation matrix Q is

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{b}{b^2+c^2} & -\frac{c}{b^2+c^2} \\ 0 & \frac{c}{b^2+c^2} & \frac{b}{b^2+c^2} \end{pmatrix}$$

In this case you have :

$$Q = 981,\ 0,\ 0<,\ 90,\ \frac{\sqrt{3}}{2},\ \frac{1}{2}=,\ 90,\ -\frac{1}{2},\ \frac{\sqrt{3}}{2}==$$

and the equation of the rotation are:

$$X = X$$

$$Y = \frac{\sqrt{3}\ Y}{2} + \frac{Z}{2}$$

$$Z = -\frac{Y}{2} + \frac{\sqrt{3}\ Z}{2}$$

The new equation of the quadric is :

$$-4 + 3\,X^2 + 2\sqrt{6}\ Y$$

The last step is to do a new translation in order to eliminate the constant term.

In order to do this, note that

$$c\,y + d = c\ Jy + \frac{d}{c}N$$

So $Y = \sqrt{\frac{2}{3}} + Y$

Options New exercise

Classification Canonical form Verification

Rotation Translation

Graph Standard Graph

Verification@"degenerate", "parabolic cylinder", "non central"D

The classification is quite good.

Options New exercise

Give a quadric Random quadric Help Example Teory
Standard graphs

GiveAQuadric@-3 - 5 x + 2 x^2 + y - 5 x y + 2 y^2 + 4 z + x z + y z - z^{2}D

Classification Canonical form Verification

The quadric matrix is:

$$A1 = \begin{pmatrix} 2 & -\frac{5}{2} & \frac{1}{2} & -\frac{5}{2} \\ -\frac{5}{2} & 2 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -1 & 2 \\ -\frac{5}{2} & \frac{1}{2} & 2 & -3 \end{pmatrix}$$

and its determinant is 0

The quadratic matrix is :

$$A = \begin{pmatrix} 2 & -\frac{5}{2} & \frac{1}{2} \\ -\frac{5}{2} & 2 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -1 \end{pmatrix}$$

and its determinant is 0

The eigenvalues of A are: $\frac{-3}{2}, 0, \frac{9}{2}$

The first step is to check if the quadric is degenerate or not

checking if the determinant of the quadric matrix is zero or not.

In this case it is zero, so the quadric is degenerate.

Now you need to compute the rank of the matrices A1 and A, and you have

rank of A = 2

rank of A1 = 2

Since the rank of A1 is 2, the quadric is central.

Moreover the rank of A is two and the eigenvalues are not all positive,

so the quadric is a pair of non coincident real planes.

Options **New exercise**

Classification **Canonical form** **Verification**

Rotation **Translation**

Graph **Standard Graph**

The given quadric is a pair of planes,whose equations are:

$-3 + x - 2 y + z$

$x + \frac{1}{2} H1 - y - zL$

Options **New exercise**

5. Conclusions

From our experience, the students comments encourage the education approach. Students are very keen on using computer-based educational supports. In particular, in view of e.g. C.A.D. applications, a "visual" approach to mathematical concepts seems the most appropriate from both the pedagogical and the experimental point of view.

The possibilities offered by Mathematica 3.0 are relevant from the educational point of view in order to produce efficient computer based tutors. It allows the realisation of training modules of high cognitive and didactic content. The package presented in this paper represents a user-friendly way to learn some fundamental concepts by oneself.

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