Computing subfields in pure trascendental extensions

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Rational function fields $K(x_1, \cdots, x_n)$ arise in various contexts within mathematics and computer science. Two examples are the invariant theory and the design of diffractive optical systems. Several questions related to subfields of $K(x_1, \cdots, x_n)$ occur in control theory in the structural identifiability problem, i.e., to check whether a differential equation model process uniquely determines the internal parameters of the process actions. Multivariate decompostion of rational functions proves useful for inverting rational functions.
The motivation of this paper on the one hand is to generalize the univariate rational function decomposition problem to the multivariate one; on the other hand, we want to give new algorithms for questions concerning unirational and rational fields. We also hope that our approach will be a step to the better understanding of the design of optical systems for computation. For this we consider the following computational problem:

Given $f_1, \cdots, f_m \in K(x_1, \cdots, x_n)$, we want to know if there exist $h_1, \cdots, h_t \in K(x_1, \cdots, x_n)$ such that $K(f_1, \cdots, f_m) \subset K(h_1, \cdots, h_t)$, is a proper algebraic extension.
And in the affirmative case, compute such h's and rational functions $g_1, \cdots, g_m \in K(y_1, \cdots, y_t)$ such that

$$f_i = g_i(h_1, \cdots, h_t) \text{ for all } i \in \{1, \cdots, m\}$$

In this talk we present a polynomial time method to solve the above problem. The algorithm is based on the computation of subfields (using blocks of decomposition) of an algebraic extension. The basic idea underlying our approach is the translation of decompositions issues to factorization over algebraic extensions and then we find minimal blocks.