Communications of the Poisson Series Processors PSPC with general scientific software.

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Extended abstract

In a previous paper [?] we described ATESAT, a software system to obtain automatically analytical theories in the artificial satellite problem. The core of ATESAT is an specialised computer algebraic system, named PSPC, used to handle a *Mathematical* object, the Poisson Series, that appears very frequently in Non-linear Mechanics and in particular in Celestial Mechanics.

A Poisson series is a multivariate Fourier series of the form

$$\sum_{i_0,\dots,i_{n-1},j_0,\dots,j_{m-1}} C^{j_0,\dots,j_{m-1}}_{i_0,\dots,i_{n-1}} x_0^{i_0}\dots x_{n-1}^{i_{n-1}} \binom{\sin}{\cos}(j_0 y_0+\dots+j_{m-1}y_{m-1});$$

this series consists of a rational or real coefficient, n polynomial variables with integer exponents and a trigonometric function of a linear combination of angular variables. The set $\mathcal{P}$ of Poisson series is a commutative algebra with a unit element. We call it series by an abuse of language, because although it may have a finite or infinite number of terms, in practice, in order to define an equivalent computational object, we need to consider only series with a finite number of terms.

Usually, high order theories in Celestial Mechanics include series with a very big number of terms. For instance, in ATESAT we manipulate around 1200000 algebraic terms of this kind of series to apply three Lie transformations (elimination of the parallax, elimination of the perigee and Delaunay

normalization) to the main problem of the artificial satellite and create a six order theory.

PSPC is a library of functions to manipulate algebraically Poisson Series. It reproduces its algebraic structure in the computer and extends the range of operations to differential and integral calculus and other more involved operations such as substitutions of variables by series, splitting series depending on certain conditions, etc. A more detailed description of PSPC can be found in [?].

Once we apply PSPC for solving a problem, we obtain as result one or more Poisson Series with a variable size depending on the problem. Usually these series must be used:

- To make a qualitative study of the results using other computer algebraic systems like *Mathematica* or other with more general tools including graphics.
- To evaluate a theory making a quantitative study of the problem by means of a compiled program using languages like C or FORTRAN.
- To use a scientific word processor, like LaTeX to write a paper with the results of the theory.

Thus, a manipulation of the results is needed to write the series in a format compatible with a software different than PSPC. An automatic translation of the series would avoid the possible errors of the manual manipulation of the series. We describe here the characteristics of the translation of the Poisson Series generated by PSPC to a format readable by *Mathematica*, C and LaTeX.

In the case of LaTeX, the problem is a simple translation between two different languages. However, when we try to use Poisson Series in a C program to evaluate it, or in a *Mathematica* session to handle it together with a more general range of functions, we must take into account different aspects than the simple translation.

Considering the big size of the series appearing in particular problems, like the ones handed by ATESAT, we need a way to write automatically an efficient C or FORTRAN code to evaluate the series. The efficiency will consist in reducing as much as possible the number of accesses to the mathematical library, minimizing the evaluation of powers and trigonometric functions and a good definition of the data structures used in the evaluation. An algorithm of Coffey & Deprit [?] was used to make this efficient code; however, the characteristics of the internal storage format of the series in PSPC makes possible to improve the efficiency of the previous algorithm in certain cases.

PSPC can collaborate with *Mathematica* in two different ways. The simplest one is the translation of the Series to the *Mathematica* format in order to use them in a *Mathematica* session as a usual *Mathematica* object. A more involved way is to create, by means of the communication protocol, MathLink (a new kernel of *Mathematica*) named PSPCLink. By means of this new kernel we will pass to PSPCLink all the operations with Poisson Series and we will pass to the kernel of *Mathematica* the rest of operations. With this tool we combine the flexibility of *Mathematica* with the efficiency of PSPC.

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