Effective computation of algebra of derivations of Lie algebras [1]

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Abstract

The knowledge of the algebra of derivations, $\mathcal{D}er(\mathfrak{g})$, of a Lie algebra $\mathfrak{g}$ allows to study many geometrical properties of $\mathfrak{g}$. Moreover, $\mathcal{D}er(\mathfrak{g})$ has also applications to other problems in many branches of science (Riemannian geometry, symmetries of differential equations, dynamical systems, quantum mechanics, theory of relativity...).

The computation of algebras of derivations for the Lie algebras is a very difficult problem. Up to now, the problem has been tried only for low dimensional Lie algebras or searching an adequate graduation of the algebra which allows to simplify the computations.

We develop a method for the computation of the algebra of derivations of Lie algebras which is adequate for the nilpotent case, specially when the nilindex is small.

Using that method we obtain the space of derivations, $\mathcal{D}er(\mathfrak{g})$, for the $(n-4)$-filiform Lie algebras of dimension n, for any n. This is the only family of nilpotent Lie algebras with nilindex 4 which is known up to now.

In this work, we show a program using of the software *Mathematica* 3.0 to determine the space of derivations of any Lie algebra and, in particular, we apply this method and this program to a special class of Lie algebras, that is, one family of $(n-4)$-filiform Lie algebras.

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