About an algorithm of T. Oaku

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Let be $f \in \mathbf{C}[x]$ with $x = (x_1, \ldots, x_n)$, and $\mathcal{A}_n$ the Weyl algebra with n variables. The polynomials $b(s) \in \mathbf{C}[s]$ such that there exists $Q \in \mathcal{A}_n[s]$ with:

$$Q(f^{s+1}) = b(s)f^s,$$

form an ideal of $\mathbf{C}[s]$, which is a principal ideal domain. The Bernstein polynomial asociated to f is this monic polynomial $b_f(s)$ of that ideal.

The proof given by Bernstein of the existence of $b_f(s)$ is not constructive, and that is why there was not effective algorithm for the calculation of the Bernstein polynomial.

In 1997, T. Oaku ("An algorithm of computing b-functions", Duke Mathematical Journal, **87**, vol. 1, 115–132) gave a method which calculate the Bernstein polynomial. This algorithm used the computation of a Gröbner basis of an ideal for an order $\prec_F$ that was not a well ordering. In order to solve that problem, the author introduced the F-homogenization of the operators, in the ring $\mathcal{A}_{n+1}[x_0]$. Then he defined a new order in that ring, the homogenization order, that was a well ordering and such that there always exists a H-Gröbner basis. Dehomogenizing that Gröbner basis he returned to $\mathcal{A}_{n+1}$ and obtained a basis for the initial order $\prec_F$. Using elimination tecniques he finally gave the Bernstein polynomial.

From this work, we deal with the problem as a Gröbner basis computation for a graded ideal in the graded algebra given by Kashiwara–Malgrange filtration. Using a new method, we define now the homogenization in the Rees algebra. This comes from "Homogenising differential operators" by F. J. Castro–Jiménez and L. Narváez–Macarro, (Prepublicaciones de la Facultad de Matemáticas, Universidad de Sevilla, **36**, 1998), permiting an ample degree of freedom in the choice of the order. Let us introduce a new order $\prec_V$, that order monomials of bigger V-order first and besides, is a elimination order in x and ∂. This new method allows to calculate just a Gröbner basis in the Rees algebra, (for a homogenization order), and that basis chives directly to the Bernstein polynomial. With this new algorithm we have a range of options for the elimination order. We finish with the calculation of non trivial examples using the new method.

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