Slopes in submodules of a free module.

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Let $\mathcal{A} = A_n(\mathbf{C}) = \mathbf{C}[X] < D >= \mathbf{C}[x_1, ..., x_n] < \partial_1, ..., \partial_n >$ be the n-th Weyl algebra, and $L : \mathbf{Q}^2 \longrightarrow \mathbf{Q}$ a linear form with possitive coeficients.

Let us consider the free module $\mathcal{A}^p$ and $\mathcal{N}$ a submodule. The L-order of an element is the maximum of the values of $L(|\beta|, \beta_1 - \alpha_1)$ over the exponents (α, β) in any component of the element. This order induces a L-filtration and an associated graduated module $gr^L(\mathcal{A}^p)$.

We will say that L is a slope of $\mathcal{M} = \mathcal{A}^p/\mathcal{N}$ if the ideal

$$\sqrt{A_L^{\mathcal{M}}} = \sqrt{Ann_{gr^L(\mathcal{A})}(gr^L(\mathcal{A}^p)/gr^L(\mathcal{N}))}$$

is not (F, V)-bihomogeneous, where F is the graduation respect to the order on D and V is the graduation of Malgrange-Kashiwara.

In this work we generalize "How to calculate the slope of a $\mathcal{D}$- module" from Assi-Castro-Granger, and "Homogenising differential operators" from Castro-Narvaez. We provide an algorithm of effective calculus of slopes as well, that we have implemented in COMMON LISP. Eventually, we use MACAULAY to calculate radicals of ideals of polynomial rings.

Some concrete examples are treated:

- Syzygy modules.
- Modules of the form $\mathcal{O}[1/f]/\mathcal{O}$ with f a surface.
- $\mathcal{A}^2/\mathcal{N}$ with $\mathcal{N} = I_1 \oplus I_2$, and each I_i a left ideal of $\mathcal{A}$ with known slopes.