Forward kinematics of parallel robots

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Abstract: The purpose of this paper is to present a state of the art in the field of the forward kinematics of parallel robots. The problem is to determine the possible pose of the end-effector of the robot being given its articular coordinates and amount to solve the non-linear system of equations obtained from the inverse kinematics. This problem is known to be one of the most challenging in the field of kinematics: it involves the resolution of a relatively large set of non-linear equations in order to determine the current pose of the end-effector. Although they are many different possible mechanical architecture for parallel robots the direct kinematics of most of them may be reduced to solving the direct kinematics for a small subset of architectures, the most important being the famous Gough platform.

This problem may be considered from different viewpoints:

1. determining the maximum number of solutions of the system either real or complex
2. determining the maximum number of real solutions of the system
3. calculating all the solutions without constraint on the computation time
4. finding an example which has exactly the maximum number of real solutions calculated for problem 2
5. if all the real solutions have been found, sort the solutions set in order to determine the current pose
6. calculating the current pose with real-time constraint

The two first problems have been addressed in the last ten years, starting with particular geometries of robots (see for example [2, 8, 9, 17, 6]) until a solutions has been given for the most general cases of Gough platform (see [13, 16, 18, 20], for which they may be up to 40 solutions.

Problem 3 has received a lot of attention and many algorithms have been proposed for particular architectures. Still for the most general case of Gough platform although specific algorithms have been proposed [11] they are not exactly satisfactory either because their computation time is still large or they have to rely on symbolic computation system to find the solutions.

Problem 4 is still an unsolved problem although a conjecture is that there will always be a robot geometry and a set of articular coordinates such that the number of real solutions of the direct kinematics will reach the maximum.

This has been shown for particular architectures [14, 15] and recently for the most general case of general Gough platform [5]. Still for many architectures the conjecture has still to be verified [6].

Problem 5 is quite important in practice: for real applications solving the direct kinematics means to determine the current, real pose of the end-effector of the robot. Consequently computing all the solutions is only the first step of the resolution of the direct kinematics. Then is it is necessary to sort the solutions in order to determine the current pose of the end-effector. Currently there is no known solution to this problem: sorting the solution will have to rely on an algorithm which will be able to determine which solution may be reached from an initial assembly mode without crossing a singularity (but this is not sufficient as different solutions may be joined without crossing a singularity as shown in [12, 10]) and without having to dismantle the robot.

Problem 6 is also very important in practice. Usually a Newton-Raphson scheme [19] is used to solve this problem, as an estimate of the solution is known, but this method has some drawbacks. The first problem in this method is to determine the parameterization of the end-effector pose (especially the orientation part) in order to have the largest convergence domain. The second drawback is that Newton method does not always converge toward the solution which is the closest to the estimate: this is a major drawback as the control of the robot heavily rely on the direct kinematics. Alternative method have been proposed: neural network [7, 21], genetic algorithms [1], interval analysis [4], improved Newton method [3] and their merits will be discussed.

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