The Center Variety of Polynomial Differential Systems

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Consider the two-dimensional differential system of the form

$$\frac{dx}{dt} = P(x, y), \qquad \frac{dy}{dt} = Q(x, y) \tag{1}$$

where P and Q are polynomials with complex coefficients and x and y are complex functions. Suppose $(0, 0)$ is a singular point of System (1). Poincaré defined the notion of a center for real vector fields in the plane. He was interested in the problem of the stability of the solar system. Dulac defined the notion of a center for complex vector fields and gave necessary and sufficient conditions for System (1) to have a center at the origin, for the case $\deg(P) = \deg(Q) = 2$. Identifying those systems with a center at the origin is the so-called *center problem.* This problem is still open for higher dimensional systems.

To solve the center problem for a given system (1), one needs to compute the so-called Lyapunov quantities, which are polynomials in the coefficients of P and Q. In this talk, we present an algorithm, using methods from computational algebra, to find all those systems with a center at the origin, such that the linear parts of P and Q form a nonsingular matrix over the complex numbers. If we restrict ourselves to real systems, our algorithm finds all the systems with a center at the origin and an axis of symmetry through the origin.