Using $\ln(|x|)$ as an Antiderivative for $1/x$ is a Bad Choice!

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Abstract

All major CAS are using ln(*x*) for their indefinite integral of $1/x$. Some will say because these systems are "complex oriented". The symbolic handheld calculator Voyage 200 (or TI-89 Titanium) from Texas Instruments gives the user the choice: when the exponential format is set to "rectangular", you get ln(*x*) for the indefinite integral of $1/x$. But, unfortunately, the exponential format defaults to "real" and we obtain $\ln(|x|)$. In single variable calculus, we teach students that two antiderivatives of a given function over an interval differ by a constant. But calculus textbooks are still using the formula $\int \frac{dx}{x} = \ln(|x|) + C$ because they want to include both cases: negative and positive *x*. There is no need for this: in the case of a definite integral, say $\int_{-3}^{-2} \frac{dx}{x}$, we lose the chance of recalling that the function $1/x$ is an odd function … or we lose a chance to introduce students to complex numbers (using log of negative numbers). In their studies, (engineering) students will also follow a Differential Equations course. They will make use of a "dsolve" command. And sometimes, in this case, some internal complex values will be necessary in order to obtain the solution of a given DE. So, here again, choosing the "real branch" is a bad choice. Finally, as far as complex analysis is concerned, the antiderivative of $1/z$ is ln(*z*) and not $\ln(|z|)$.

Using technology becomes an opportunity to question textbooks: this will be the main goal of this talk.