Algebra + Geometry ==> Differential Equation Solving

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Consider an autonomous algebraic ODE of the form $F(y, y') = 0$, where F is a bivariate polynomial. We can think of F as defining a plane algebraic curve. If this curve admits a rational parametrization, then we can determine whether the ODE has a rational general solution. Based on degree bounds for such parametrizations by Sendra and Winkler, Feng and Gao have described an algorithm for this problem.

Here we extend this investigation to the case of a non-autonomous algebraic ODE of the form $F(x, y, y') = 0$. The tri-variate polynomial $F(x, y, z)$ defines an algebraic surface, which we assume to admit a rational parametrization. Based on such a parametrization and on knowledge about a degree bound for general rational solutions, we can determine the existence of a general rational solution, and, in the positive case, also compute one. This method depends crucially on the determination of rational invariant algebraic curves. We also relate rational general solutions of algebraic ODEs to rational first integrals.