An Automatic Deduction Environment for Dynamic Geometry Based on the Open Source Computer Algebra System Sage

Francisco Botana
Departamento de Matemática Aplicada I, Universidad de Vigo, Campus A Xunqueira, 36005 Pontevedra, Spain
fbotana@uvigo.es

Miguel A. Abánades
CES Felipe II, Universidad Complutense de Madrid, 28300 Aranjuez, Spain
abanades@ajz.ucm.es

Abstract

A Dynamic Geometry System (DGS) is an application that allows the exact on-screen drawing of (generally) planar geometric diagrams and its manipulation by mouse dragging certain elements making all other elements to automatically self adjust to the changes. This is also known as Interactive geometry.

From the beginning, DGS have been the paradigm of new technologies applied to Math education. Being able to produce a great number of examples of a configuration has many times been taken as a substitute for a formal proof in what has come to be known as a *visual proof*. Questions have been raised on the influence of this use of DGS on the development of the concept of proof in school curricula [5]. This is a symptom of the incompleteness of general DGS relative to further manipulation of configurations. Although most DGS considered come equipped with some property checker, their numeric nature does not really provide a sound substitute for a formal proof.

To compensate the computational limitations of DGS, two main approaches have been taken to add symbolic capabilities to DGS. Some systems incorporate their own code to perform symbolic computations (e.g. [3]), while others choose to reuse existing Computer Algebra Systems (e.g. [2, 1, 8, 10]). A prototype strictly based on open source tools that offers a solution in this latter direction is presented. If we take open as a synonym for accessible, we can not find fields where this concept is more relevant than Education, where universal access should be the leading principle, and Mathematics, where public scrutiny is at its very foundations.

More concretely, we present a symbolic tool that provides robust algebraic methods to handle automatic deduction tasks for a dynamic geometry construction. The prototype has been developed as two different worksheets for the open source computer algebra system Sage [9], corresponding to two different ways of coding a geometric construction. In one worksheet, diagrams constructed with the open source dynamic geometry system GeoGebra [4] are accepted. In this worksheet, Groebner bases are used to either compute the equation of a geometric locus in the case of a locus construction or to determine the truth of a general geometric statement included in the GeoGebra construction as a boolean variable. In the second worksheet, locus constructions coded using the common file format for dynamic geometry developed by the Intergeo project [6] are accepted for computation.

Submitted to:
ACA 2011

References

[1] F. Botana & J. L. Valcarce (2002): *A dynamic-symbolic interface for geometric theorem discovery*. *Computers and Education* 38, pp. 21–35.

[2] J. Escribano, F. Botana & M. A. Abnades (2010): *Adding Remote Computational Capabilities to Dynamic Geometry Systems*. *Mathematics and Computers in Simulation* 80, pp. 1177–1184.

[3] X. S. Gao, J. Z. Zhang & S. C. Chou (1998): *Geometry Expert*. Nine Chapters, Taiwan.

[4] GeoGebra: Available at `http://www.geogebra.org`.

[5] C. Hoyles & K. Jones (1998): *Proof in Dynamic Geometry Contexts*, chapter Perspectives on the teaching of Geometry for the 21st Century, pp. 121–128. Kluwer, Dordrecht.

[6] Intergeo: Available at `i2geo.net/`.

[7] J. Richter-Gebert & U. Kortenkamp (1999): *The Interactive Geometry Software Cinderella*. Springer, Berlin.

[8] E. Roanes-Lozano, E. Roanes-Macías & M. Villar (2003): *A bridge between dynamic geometry and computer algebra*. *Mathematical and Computer Modelling* 37, pp. 1005–1028.

[9] Sage: Available at `http://www.sagemath.org`.

[10] D. Wang (1996): *Automated Deduction - Cade-13*, chapter GEOTHER: A geometry theorem prover, pp. 166–170. *LNCS* 1104/1996, Springer.