Computation of Hilbert Schemes

M. Albert[1]

[1] *University of Kassel, Germany, albert@mathematik.uni-kassel.de*

Hilbert schemes are a basic topic in Algebraic Geometry [1, 2]. Methods related to Gröbner bases are fundamental here, as Hartshorne's proof of the connectedness of Hilbert schemes via generic initial ideals demonstrates [3]. For many purposes, the explicit construction of Hilbert schemes is important. Classical approaches lead to prohibitively large equations. Newer ideas like Gröbner strata [4, 5] lead to improvements, but do not provide open covers. Despite all advances, it has remained a great challenge to construct Hilbert schemes even for very small Hilbert polynomials.

A novel approach replacing Gröbner bases by *J*-marked bases [6, 7] was proposed by the group of Margharita Roggero. For every strongly stable ideal J, one obtains here a larger family which corresponds to an open subscheme and which can be described by equations of low degree.

We have implemented several of the new algorithms in the computer algebra system COCOALIB [8]. In the talk we are going to explain these algorithms and give some ideas to improve these algorithms. In addition to that we present some first practical experience which we have made during first computations. Furthermore we report some experiences which we have made when we tried to parallelize these algorithms.

References

[1] A. Grothendieck. Techniques de construction et théorèmes d'existence en géométrie algébrique. IV. Les schémas de Hilbert. In *Séminaire Bourbaki, Vol. 6*, pages Exp. No. 221, 249–276. Soc. Math. France, Paris, 1995.

[2] E. Sernesi. *Deformations of algebraic schemes*, volume 334 of *Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences]*. Springer-Verlag, Berlin, 2006.

[3] R. Hartshorne. Connectedness of the Hilbert scheme. *Inst. Hautes Études Sci. Publ. Math.*, (29):5–48, 1966.

[4] Jerome Brachat, Paolo Lella, Bernard Mourrain, and Margherita Roggero, *Extensors and the Hilbert scheme*, Annali della Scuola Normale Superiore di Pisa, Classe di Scienze (2015), Article in Press, doi:10.2422/2036-2145.201407_003.

[5] Mathias Lederer, *Gröbner strata in the Hilbert scheme of points*, J. Commut. Algebra **3** (2011), no. 3, 349–404.

[6] C. Bertone, P. Lella and M. Roggero. A Borel open cover of the Hilbert scheme. *J. Symbolic Comput.*, 53:119–135, 2013.

[7] F. Cioffi and M. Roggero. Flat families by strongly stable ideals and a generalization of Gröbner bases. *J. Symbolic Comput.*, 46(9):1070–1084, 2011.

[8] Abbott, J., Bigatti, A.: CoCoALib: a C++ library for doing computations in commutative algebra (2016), `http://cocoa.dima.unige.it/cocoalib`