Nœther Normalization and Involutive Bases

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In this talk, we present a relationship between Nœther normalization and involutive bases. Based on a new definition of Nœther position called *D-quasi stability* [1] we present an efficient algorithm based on the algorithm described by Seiler in [2] (to transform an ideal into quasi stable position) to find, for a given homogeneous ideal, a linear change of variables which transforms the ideal into Nœther position. For this purpose, the type of linear changes we apply is the elementary linear changes, i.e. at each iteration we make a linear change of the form $x_k \mapsto x_k + ax_\ell$ where the pair (x_k, x_ℓ) presents an obstacle for being in Nœther position and a is a random number. We have implemented the described algorithm in MAPLE and illustrate its efficiency via a set of benchmark polynomials. In this direction, we shall mention the work [3] due to Robertz in which he applied the cone decomposition of the given ideal using Janet division to obtain also an sparse linear change of variables for the Nœther normalization. Remark that the criterion to get Nœther normalization described in [3] is *not* equivalent to Nœther position, however, our new criterion is equivalent and this may lead to achieve a more sparse linear change of variables.

References

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