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Analyzing the "Calculator Effect" of Different Kinds of Software for School Arithmetics and Algebra

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We investigate here the possibility of getting the answer and solution of arithmetic and algebra tasks from outside pieces of software when the intended solution environment is paper and pencil or some teacher-controlled software system.

Introduction of numeric calculators gave students the possibility to perform numeric calculations with decimal numbers automatically, asking from the device step by step execution of operations. Automatic calculation enables solving problems that contain more bulky numeric data. But it can also undermine learning if students can get the answers from the screen instead of doing calculation exercises. Using the classical numeric calculator, the student also gets results of all intermediate steps. Thereby the student should know the order of operations in the expression. However, from some level, this is a routine thing. More recent calculators enable also entering the numeric expression and calculation of its value in one step.

The effect gets a new dimension in algebraic tasks that require from the student knowledge of the solution algorithm or invention of own solution. Usually the student should here submit a step by step solution. Algorithmic and creative tasks will be completely trivialized if the software provides automatically not only the answer but also the solution steps.

Computer algebra systems expanded the automated solution capacity from the tasks of arithmetic to algebra: operations with fractions, expanding and factorizing of polynomials, solving different equations, etc. But here the student can usually get only the answer. The current CAS's do not output the solution and often even do not have detailed commands for getting the intermediate results.

The author of this paper realised the need to think about misuse effects when participating in two activities connecting Mathematics teaching and computers:

1) Addition of school-style step by step solutions to the output of CAS,

2) Compiling a workbook (for teaching programming in secondary school) that contains programming tasks extracted from School Mathematics.

Both kinds of applications give the student new options to learn Mathematics but also options to get the answer or solution from the computer. Answers and solutions of arithmetic and algebraic tasks can also be produced by other existing kinds of software:

3) Spreadsheets,

4) Available lightweight drill environments for arithmetic and algebra (search for 'operations with fractions', 'linear equation', etc),

5) More sophisticated solution environments that have step by step solution dialog (like MathXpert [1] , Aplusix [2] and T-algebra [3]).

6) "Algebra calculators" (see [4] for some overview).

Spreadsheets are not designed for working with algebraic expressions. We can speak here mainly about calculation of the values of numeric expressions that can contain elementary functions. In our practical use we think that spreadsheets do decimal calculations. But this is not completely true; spreadsheets cheat us a bit. If we enter in Excel '$= 1/3$' then we get $0.3\ldots300\ldots$ with up to 15 digits 3. But if we add three such numbers then we get 1. This means that Excel actually records $1/3$ and then uses computer algebra. At the same time, if we compare this $1/3$ with decimal fraction $0.3\ldots300\ldots$ then they are equal! But the sum of three decimal values is $0.9\ldots90\ldots$ and not equal to 1. The same scenario gives slightly different results in LibreOffice. The manuals do not describe this mixture of decimal and algebraic calculations. It is also clear that this description would be too complex for the school. The conclusion seems to be that we should warn about strange effects of spreadsheet calculations.

Small drill environments are available for many types of exercises of Basic School Arithmetic and Algebra. Mostly, they ask from the student just entry of the final answer. But some of them also request results of certain intermediate stages of the solution algorithm. Many such environments offer demo solutions of some spectrum of fixed tasks. But this does not have calculator effect because usually they do not enable entering user-provided exercises.

We cannot tell very much in general about dedicated solution environments. Their solution dialogs are usually quite precise in prescribing possible work scenarios. In order to use an environment in the role of algebra calculator, it should have an appropriate task type and produce output of full solution or step hints. For example, the solver of MathXpert [1] provides solutions of large variety of task types of Algebra and Calculus. Again, the student needs here the right to enter initial expressions of the tasks.

There are a few dozen programs designed specifically for doing students' homework (producing solutions with necessary explanations). A search for 'algebra calculator' returns several pages of web links to such programs. Some of them are free, some have daily limitations for free use, and some are commercial with moderate price (up to 10 dollars per month and about 50 dollars per year). There exists even an Algebra Calculators Guide: "144 Calculators Separated by Skill Level" [4]. This Guide is very positive and does not speak about the quality of solutions. The author's impression is that several solvers implement textbook algorithms without any intelligence. For some task types this is sufficient but often the textbook algorithms assume that some final choices are made by the student. For example, in case of the substitution method, the student may be expected to decide what variable to isolate and from what equation. Several 'calculators' always isolate the first variable from the first equation. The teacher has problems when a student submits a series of brute-force solutions (written without any mistakes!).

Consider now the situation before introduction of step by step solutions in CAS. There are online step by step solvers that solve most of the tasks of School Algebra. For some task types (where textbook algorithms leave some choices to the student) the quality of automated solutions is low. Some less common task types can be missing. Some topics are ignored (mixed numbers or doing operations with them). However, availability of solutions in educational CAS (for example, in Geogebra) will not change the situation with cheating very much.

For elaboration of step by step solutions the CAS needs addition of detailed commands for school-level solution steps. After that it is possible to implement textbook algorithms and solution methods for most important tasks that have no textbook algorithm (for example, factorization). Mathematics traditions of some countries also require the use of mixed numbers. In the first years, the CAS solutions will most likely have the same imperfections

as the current algebra calculators. But even later the presented solutions cannot conform to all necessary combinations of country-, textbook- or teacher-specific requirements. Weaker students are not able to translate the output of CAS in the required "language" and/or explain the solution steps.

Many school arithmetic and algebra tasks can be converted to programming tasks: long multiplication or division, reducing fractions, multiplication of polynomials. Creating pieces of computer algebra can be an interesting programming exercise but can also give new insights into the algorithmic side of Mathematics. For routine tasks it does not replace exercising with something easier. The situation can be different when we come to the most original tasks in textbooks. A task of replacing stars with given numbers can lead to an interesting logical journey. But brute-force solution search can require only some 3 or 5 embedded loops with checking another 3 or 5 conditions in its body. This level of programming is completely feasible for stronger students of middle grades. Once written, the program can be easily modified for many similar tasks. The speed of computers can completely remove many of the interesting combinatorial tasks from the basic school Mathematics textbooks. We should think how to protect the more original tasks.

Keywords: Calculator effect, Computer algebra, School Mathematics

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References

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