Visualizations of the nondominated set and the efficient set in multicriteria optimization problems using Mathematica

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Multicriteria optimization also known as multicriteria programming is a sub-discipline of operation research. It is taught students in framework of such academic courses as for example: Operation research, Multiobjective optimization, Optimization methods or Mathematical programming. Multicriteria optimization problem has a general form:

\[ f(x) = (f_1(x), f_2(x), \ldots, f_k(x)) \rightarrow \text{min/max} \]
subject to \( x \in X, X \subset \mathbb{R}^n \)

where \( x \) is a decision variable vector, \( X \) is a feasible set in decision space \( \mathbb{R}^n \), \( (f_1(x), f_2(x), \ldots, f_k(x)) \) is a criterion vector and min or max are understood in accordance with the partial order \( P \) in criterion space \( \mathbb{R}^k \). We define: a feasible set \( Y \) in criterion space as the image of the set \( X \) under \( f = (f_1, f_2, \ldots, f_k) \), the nondominated set \( Y_N = \{ y \in Y : \text{there is not } y' \in Y \text{ with } y' P y \} \) and the efficient set \( X_E = \{ x \in X : f(x) \in Y_N \} \). Many academic books contain visualizations of sets \( X, Y, Y_N, X_E \) for some linear functions \( f : \mathbb{R}^2 \rightarrow \mathbb{R}^2 \). It would be more difficult but didactically useful to present these sets also for functions \( f : \mathbb{R}^2 \rightarrow \mathbb{R}^3 \) and \( f : \mathbb{R}^3 \rightarrow \mathbb{R}^3 \). It would rather require computer support using for example CAS programs. In this talk we would like to present a few didactic visualizations of sets \( X, Y, Y_N, X_E \) for some functions \( f : \mathbb{R}^2 \rightarrow \mathbb{R}^3 \) and \( f : \mathbb{R}^3 \rightarrow \mathbb{R}^3 \) using Mathematica.

Keywords: multicriteria optimization, multicriteria programming, didactics of mathematics, mathematics education, CAS, Mathematica

References


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