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Computations of Gröbner-Shirshov Bases and Normal Forms for Affine Weyl Groups $\widetilde{B_n}, \widetilde{C_n}$ and $\widetilde{D_n}$ Using Mathematica

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Gröbner and Gröbner-Shirshov bases theories are generating increasing interest since its usefulness in providing computational tools and in giving algebraical structures which are applicable to a wide range of problems in mathematics, engineering, science and computer science. In particular Gröbner and Gröbner-Shirshov bases theories are powerful tools to deal with the normal form, word problem, embedding problem, extentions of algebras, Hilbert series, etc. The importance of Gröbner-Shirshov bases is that they can be computed.

Gröbner-Shirshov bases and reduced form of the elements were already found for the Coxeter groups of type A_n, B_n and D_n in [1]. They also proposed a conjecture for the general form of Gröbner-Shirshov bases for all Coxeter groups. In [3] , an example was given an to show that the conjecture is not true in general. The Gröbner-Shirshov bases of the other finite Coxeter groups are given in [4] and [5]. This paper is another example of finding Gröbner-Shirshov bases for groups, defined by generators and defining relations.In [6], we calculated Gröbner-Shirshov Basis and reduceds words for $\widetilde{A_n}$. In this talk, we deal with the affine Weyl group $\widetilde{B_n}, \widetilde{C_n}$ and $\widetilde{D_n}$ which is an infinite Coxeter groups.

The main aim of this talking is to find the reduced Gröbner-Shirshov bases of $\widetilde{B_n}, \widetilde{C_n}$ and $\widetilde{D_n}$ and applications classify all normal forms for the affine Weyl groups of $\widetilde{B_n}, \widetilde{C_n}$ and $\widetilde{D_n}$, using defining relations. We use the following strategy for solving the problem:

Although Gröbner bases algorithms implemented in Computer Algebra systems, there is no efficient Computer Algebra packages to compute Gröbner-Shirshov bases. Because of non-commutative structure, it is not easy to find Gröbner-Shirshov bases. We wrote a program in Mathematica to find Gröbner-Shirshov bases of $\widetilde{B_n}, \widetilde{C_n}$ and $\widetilde{D_n}$ for small n's. Then we generalize this set to any positive integer n, we called it R'. After that using the algorithm of elimination of leading words with respect to the polynomials in R', all the words in the groups $\tilde{B_n}, \tilde{C_n}$ and $\tilde{D_n}$ are reduced to the explicit classes of words for small n's with help of Mathematica. These sets are sets of normal forms of $\widetilde{B_n}, \widetilde{C_n}$ and $\widetilde{D_n}$. We also generalize these normal forms sets to any positive integer n's. To count the number of the elements of these sets, we get the correspondences of between normal forms and permutation forms. Using Mathematica, we calculated the permutation forms of the words. Then using combinatorial techniques, we compute the number of all normal forms with respect to these classes by means of generating functions. These generating functions turn out to be same with the well known Poincaré polynomials of the affine Weyl groups $\widetilde{B_n}, \widetilde{C_n}$ and $\widetilde{D_n}$. Therefore, by Composition-Diamond Lemma the functions in R' form Gröbner-Shirshov bases. for the affine Weyl groups $\widetilde{B_n}, \widetilde{C_n}$ and $\widetilde{D_n}$. Furthermore, one can easily see that these bases are in fact the normal forms Gröbner-Shirshov bases.

Keywords: Affine Weyl Groups, Gröbner-Shirshov Basis, Composition-Diamond Lemma, q-binomials, Basic Partitions

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References

[1] L.A. Bokut and L.S. Shiao, *Gröbner-Shirshov bases for Coxeter groups*, Comm. Algebra **29 (9)**, (2001), 4305-4319.

[2] A. BJORNER; F. BRENTI, *Combinatorics of Coxeter groups*, Graduate Texts in Mathematics, **231**, Springer-Verlag New York, 2005.

[3] Y.CHEN; C. LIU, *Gröbner-Shirshov bases for Coxeter groups I*, arXiv:0910.0096v1.

[4] D. LEE, *Gröbner-Shirshov bases and normal forms for the coxeter groups E_6 and E_7*, Advances in Algebra and Combinatorics, World Scientific,243–255 (2008), .

[5] O. SVECHKARENKO, *Gröbner-Shirshov bases for the Coxeter group E_8*, Master Thesis, Novosibirsk State University, 2007.

[6] E. YILMAZ; C. ÖZEL; U. USTAOĞLU, *Gröbner-Shirshov basis and reduced words for the affine Weyl group $\tilde{A_n}$*, Journal of Algebra and Its Applications,**13** (6), (2014).

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