SPECIAL SESIONS

Applications of Computer Algebra - ACA2018

June 18–22, 2018

Santiago de Compostela, Spain

S5

Computer Algebra for Dynamical Systems and Celestial Mechanics

Tuesday

Tue 19th, 10:30 - 11:00, Aula 10 – Werner M. Seiler:
On the Numerical Analysis and Visualisation of Implicit ODEs

Tue 19th, 11:30 - 12:00, Aula 10 – Werner M. Seiler:
Singular Initial Value Problems for Quasi-Linear Ordinary Differential Equations

Tue 19th, 12:00 - 12:30, Aula 10 – Alexander Perminov:
The construction of averaged semi-analytical planetary motion theory up to the third degree of planetary masses by means CAS Piranha

Tue 19th, 12:30 - 13:00, Aula 10 – Victor Edneral:
Local and Global Properties of ODEs

Tue 19th, 15:30 - 16:00, Aula 10 – Victor Edneral:
Nonlinear Oscillations of a Spring Pendulum at the 1 : 1 : 2 Resonance by Normal Form Method

Tue 19th, 16:00 - 16:30, Aula 10 – Vasilii Duzhin:
Schutzenberger transformation on the three-dimensional Young graph

Tue 19th, 16:30 - 17:00, Aula 10 – Aleksandr Mylläri:
On the estimation of complexity of trajectories in the equal-mass free-fall three-body problem

Tue 19th, 17:30 - 18:00, Aula 10 – Alexey Rosaev:
The modeling of the effect of velocity of breakup in osculating orbital elements of the young asteroid family

Tue 19th, 18:00 - 18:30, Aula 10 – Alexander Batkhin:
Searching for periodic solutions with central symmetry in Hill problem

Organizers

Victor Edneral:
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Aim and cope

Celestial Mechanics and Dynamical Systems are traditional fields for applications of computer algebra. Computer algebra methods play a fundamental role in the treatment of concrete problems and applications. Computer algebra applications include nontrivial use of existing systems Maple, Mathematica, Singular etc. and the development and implementation of new algorithms, and specialized packages. The session will bring together specialists from diverse areas: differential equations, dynamical systems and computer algebra. Expected topics of presentations include (but are not limited to):

- Stability and bifurcation analysis of dynamical systems
- Construction and analysis of the structure of integral manifolds
- Symplectic methods
- Symbolic dynamics
- Celestial mechanics and stellar dynamics. N-body problem, KAM theory
- Specialized computer algebra software for celestial mechanics
- Normal form theory and formal integrals
- Deterministic chaos in dynamical systems
- Families of periodic solutions
- Perturbation theories and reductions
- Exact solutions and partial integrals
- Analysis and blow-ups of non-elementary stationary points

- Analysis of singularities: geometry and topology
- Integrability and nonintegrability, algebraic invariant sets and Darboux integrability
- Discrete Dynamical Systems and ergodic theory
- Topological structure of phase portraits and computer visualization

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On the Numerical Analysis and Visualisation of Implicit Ordinary Differential Equations

Elishan Braun[1], **Werner M. Seiler**[2], **Matthias Seiß**[2]

We discuss how the geometric theory of differential equations [4] can be used for the numerical integration and visualisation of implicit ordinary differential equations, in particular around singularities of the equation [3]. The Vessiot theory [2] automatically transforms an implicit differential equation into a vector field distribution on a manifold and thus reduces its analysis to standard problems in dynamical systems theory like the integration of a vector field and the determination of invariant manifolds. For the visualisation of low-dimensional situations we adapt the streamlines algorithm of Jobard and Lefer to 2.5 and 3 dimensions. A concrete implementation in Matlab is presented [1].

Keywords: Implicit ordinary differential equations, Vessiot distribution, jet bundles, singular points, invariant manifolds

References

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Applications of Computer Algebra – ACA2018
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Singular Initial Value Problems for Quasi-Linear Ordinary Differential Equations

Werner M. Seiler[1], Matthias Seiß[1]

We discuss existence, non-uniqueness and regularity of solutions of initial value problems for quasi-linear ordinary differential equations where the initial condition corresponds to an impasse point [4] of the equation. With a differential geometric approach [1, 3], we reduce the problem to questions in dynamical systems theory. As an application, we discuss in detail second-order equations of the form $g(x)u'' = f(x, u, u')$ with an initial condition imposed at a simple zero of g. This generalises results by Liang [2] and also makes them more transparent via our geometric approach.

Keywords: Quasi-linear ordinary differential equations, geometric theory, initial value problem, existence and (non-)uniqueness of solutions, regularity

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Applications of Computer Algebra – ACA2018
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The construction of averaged semi-analytical planetary motion theory up to the third degree of planetary masses by means CAS Piranha

Alexander Perminov[1], Eduard Kuznetsov[1]

The study of planetary systems orbital evolution is one of important problems of Celestial mechanics. In this work authors present the algorithm for the construction of the averaged semi-analytical motion theory up to the third degree of the small parameter for the case of planetary system with four planets. In this case the small parameter is the ratio of sum of planetary masses to the mass of the star.

The Hamiltonian of four-planetary problem is written in Jacobi coordinates and it is expressed into the Poisson series in elements of Poincare's second system. It allows sufficiently simplifying an angular part of the series expansion. In this case only one angular element – mean longitude, is defined.

The averaged Hamiltonian and the motion equations in averaging elements are constructed by Hori-Deprit method as the series in the small parameter and all orbital elements. The transformation between osculating and averaged orbital elements is performed by using of the functions for the change of variables. The using of the averaged motion equations allows sufficiently increase time step of the next numerical integration.

All analytical manipulations are performed by using of computer algebra system Piranha [1]. The author of Piranha system is Francesco Biscani from Max Plank Institute for Astronomy in Heidelberg, Germany. Piranha is echeloned Poisson series processor. It is developing C++ code with Python interface for analytical calculations with polynomials, Poisson series and echeloned Poisson series.

Orbital elements and masses are kept in the series expansions as symbol variables. It should be noted that series numerical coefficients are kept as rational numbers with arbitrary precision for the elimination of rounding errors.

The terms with the first order of the small parameter in the averaged Hamiltonian is constructed up to 8-th degree of eccentric and oblique Poincare elements. The second order terms is constructed up to 6-th degree and the third order terms – up to 2-nd degree of eccentric and oblique Poincare elements. It allows to get high precision motion equations for giant planets of Solar system and various extrasolar systems also. The algorithms of the expansion into the Poisson series and the construction of motion equations are presented in this work.

The results of numerical integration of the motion equations for the Sun – Jupiter – Saturn – Uranus – Neptune's system on a time interval of 10 billion years is considered. It is performed by Everhart method of 15-th order. The motion of the planets

has an almost periodic character. The orbital eccentricities and inclinations save small values over whole time of the integration. The comparison with numerical theories is given.

The study was funded by RFBR according to the research project no. 18-32-00283 and the Government of the Russian Federation (Act no. 211, agreement no. 02.A03.21.0006).

Keywords: CAS Piranha, echeloned Poisson series processor, four-planetary problem, semi-analytical motion theory, Hori-Deprit method, Jacobi coordinates, second system of Poincare elements

References

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Applications of Computer Algebra – ACA2018
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Local and Global Properties of ODEs*

Victor Edneral[1], Valery Romanovski[2]

We consider autonomous planar systems of ordinary differential equations with a polynomial nonlinearity. These systems are resolved with respect to derivatives and can contain free parameters. To study local integrability of the system near each stationary points, we use an approach based on Power Geometry[1] and on the computation of the resonant normal form[2, 3]. For the pair of concrete planar systems[4] and[5], we found the complete set of necessary conditions on parameters of the system for which the system is locally integrable near each stationary points. The main idea of this report is in the hypothesis that if for each fixed set of parameters such that all stationary points of the equation are centers then this system has the global first integral of motion. So from some finite set of local properties we can obtain a global property. But if the system has some invariant lines or separatists, this first integral can exist only in the part of the phase space, where center points take place.

Keywords: Local Integrability, Global Integrability

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Applications of Computer Algebra – ACA2018
Santiago de Compostela, June 18–22, 2018

Nonlinear Oscillations of a Spring Pendulum at the 1 : 1 : 2 Resonance by Normal Form Method*

Victor Edneral[1], Alexander Petrov[2]

Nonlinear spatial oscillations of a material point on a weightless elastic suspension are considered. The frequency of vertical oscillations is assumed to be equal to the doubled swinging frequency (the 1 : 1 : 2 resonance) [1]. In this case, vertical oscillations are unstable, which leads to the transfer of the energy of vertical oscillations to the swinging energy of the pendulum. Vertical oscillations of the material point cease, and, after a certain period of time, the pendulum starts swinging in a vertical plane. This swinging is also unstable, which leads to the back transfer of energy to the vertical oscillation mode, and again vertical oscillations occur. However, after the second transfer of the energy of vertical oscillations to the pendulum swinging energy, the apparent plane of swinging is rotated through a certain angle. These phenomena are described analytically by the normal form method[2].

Keywords: Pendulum, Resonance, Normal form method

References

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Applications of Computer Algebra – ACA2018
Santiago de Compostela, June 18–22, 2018

On the estimation of complexity of trajectories in the equal-mass free-fall three-body problem

Mylläri Aleksandr[1], **Mylläri Tatiana**[1], **Myullyari Anna**[2], **Vassiliev Nikolay**[3]

We study complexity of trajectories in the equal mass free-fall three-body problem. We construct numerically symbolic sequences using different methods: close binary approaches, triple approaches, collinear configu- rations and other. Different entropy estimates for individual trajectories and for a system as a whole are compared.

Keywords: Three-Body Problem, Symbolic Dynamics, Entropy

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Applications of Computer Algebra – ACA2018
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Schutzenberger transformation on the three-dimensional Young graph

Vasilii Duzhin[1], Nikolay Vassiliev[1,2]

The Schutzenberger transformation on Young tableaux, also known as "jeu de taquin", was introduced in [1]. This transformation allows to solve different problems of enumerative combinatorics and representation theory of symmetric groups. Particularly, it can be used to calculate the Littlewood-Richardson coefficients [2].

It is known [3] that a limit distribution of Plancherel probabilities on the front of large Young diagrams of size $n, n \to \infty$ has the following density function known as semicircle distribution:

$$d\mu(u) = \frac{\sqrt{4-u^2}}{2 \cdot \pi},$$

where u is one of Vershik-Kerov coordinates: $u = \frac{x-y}{\sqrt{n}}$. Later it was proved [4] that the coordinates of Schutzenberger path ends are distributed according to the semicircle distribution as well.

However, there are no known limit distribution function of the coordinates of three-dimensional Schutzenberger path ends. Moreover, there are no known 3D analogues of the central Plancherel process and RSK correspondence. In this work we made an attempt to fill this gap by conducting some numerical experiments on the three-dimensional Young graph.

Also we considered a special randomized variant of the Schutzenberger transformation. It was found that this approach can be used to calculate the co-transition probabilities on the Young graph, which in turn gives a possibility to calculate the ratios of dimensions of 3D Young diagrams. Note that the exact dimensions of 3D Young diagrams cannot be calculated directly.

Keywords: Young tableaux, Young diagrams, Schutzenberger transformation, Jeu de taquin, Limit shape

References

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Applications of Computer Algebra – ACA2018
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The modeling of the effect of velocity of breakup in osculating orbital elements of the young asteroid family

Rosaev A.[1]

Asteroid families are groups of minor planets that have a common origin in breakup events. The very young compact asteroid clusters (VYF) with age smaller than 1 Myr allow us to study impact process and nonlinear dynamics. In previous our paper [1] we had noted dependence between dΩ and dϖ for Datura family but had not explained it. Additionally, we find other dependences between angular elements dω and dϖ in some other very young asteroid family. Vokrouhlicky et al. [2] have given explanation but their model cannot proper explain value of observed slope. In this paper we test the hypothesis of the primordial origin of the observed dependences at the epoch of the cluster formation. The implicit dependences of the orbital elements on breakup velocity components are studied with Maple. As a result, the dependences, similar to observed, obtained at specific values of breakup velocities.

Keywords: asteroid family, orbital evolution, breakup velocity

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Applications of Computer Algebra – ACA2018
Santiago de Compostela, June 18–22, 2018

Searching for periodic solutions with central symmetry in Hill problem*

Alexander Batkhin[1],

We consider Hill problem (see [1, 2]) describing the motion of massless body near the minor of two active masses as a regular perturbation of Kepler problem in uniformly rotating (sinodical) frame. It makes possible to apply the classical method of Hamiltonian normal form [3] for searching generating solutions of families of periodic orbits. The essential difference of the Hill problem from the well-known Restricted Three Body Problem (RTBP) is that canonical equations of motions are invariant under the group of linear transformations of the extended phase space with generators:

$$\Sigma_1 : (t, x_1, x_2, y_1, y_2) \to (-t, x_1, -x_2, -y_1, y_2)\,,$$
$$\Sigma_2 : (t, x_1, x_2, y_1, y_2) \to (-t, -x_1, x_2, y_1, -y_2)\,,$$

This fact allows two state that the set of periodic solutions can be divided into following subsets:

- asymmetric orbits, which change under any transformation;
- singly symmetric orbits, which are invariant under transformation Σ_1 or Σ_2;
- centrally symmetric orbits, which are invariant under composition $\Sigma_{12} \equiv \Sigma_1 \circ \Sigma_2$ only;
- doubly symmetric orbits, which are invariant under any transformation.

Earlier [2, 4], periodic solutions with any type of symmetry but central were computed. This work is an attempt to find generating solutions with central symmetry and to continue them into periodic solutions of the Hill problem.

Just now the following steps are realized:

1. generalized Hill problem Hamiltonian with small parameter ε is rewritten in Delaunay variables;
2. the procedure of invariant Hamiltonian normalization up to the second order is applied;

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3. the condition on existing of generating solutions can be written in the form

$$F(e, p, q) \sin k\varpi = 0,$$

where e is an eccentricity, ϖ is an argument of the pericenter, $p+q \in \{1, 2, 4\}$. Function $F(e, p, q)$ is smooth for $e \in [0; 1)$. The parameter k equals to 2 for $p + q = 1$ or 2, equals to 4 for $p + q = 4$.

The specific of the generalized Hill problem leads to the evidence that it is possible to successfully continue such generating solutions into the Hill problem periodic orbits, which major semi-axis a is less then 1, or $p, q \in \mathbb{N}$. For centrally symmetric generating solutions it is possible if $p > 2$, p is odd, and $\varpi \neq k\pi/4$, $k \in \mathbb{N}$. A suitable for continuation generating solution corresponds to the only values $p = 3$ and $e \approx 0.8525432355$.

Keywords: Hill problem, periodic solution, symmetry

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