SPECIAL SESIONS

Applications of Computer Algebra - ACA2018

June 18–22, 2018

Santiago de Compostela, Spain

S12

Numerical Differential and Polynomial Algebra

Friday

Fri 22nd, 11:30 - 12:00, Aula 9 – Dmitry Lyakhov:
Symbolic-numeric methods for simulation of cosserat rods

Fri 22nd, 12:00 - 12:30, Aula 9 – Zahra Mohammadi:
A symbolic-numeric method to determine symmetry of approximate differential equations

Fri 22nd, 12:30 - 13:00, Aula 9 – Greg Reid:
Challenges in Numerical Differential Algebra

Organizers

Greg Reid:
Western University
Ontario, Canada

Zahra Mohammadi :
Western University
Ontario, Canada

Aim and cope

The aim of this session is bring together researchers and practitioners working with systems of polynomials and also those working with systems of polynomially nonlinear dif- ferential equations. A particular emphasis of our session is on approximate methods for such systems. There have been many recent developments which yield homotopy based methods for determining approximate points on solution components of such systems. The session will also encour- age contributions on the much less developed area of ap- proximate differential algebra, fundamentally important in applications to dynamical models. We invite participants in both theory and applications to this session. This session has an overlap with the session Computational Differential and Difference Algebra.Expected topics of presentations include (but are not limited to):

- Numerical Polynomial Algebra
- Approximate Differential Algebra
- Numerical homotopy methods for witness points of poly- nomial systems
- Approximate geometric involutive differential systems
- PDAE and DAE and their applications
- Numerical methods for approximate critical points of real polynomial systems

Applications of Computer Algebra – ACA2018
Santiago de Compostela, June 18–22, 2018

Symbolic-numeric methods for simulation of cosserat rods

Dmitry Lyakhov [1]

We derive a combined analytical and numerical scheme to solve differential Kirchhoff system. Here the object is to obtain an accurate as well as an efficient solution process. Purely numerical algorithms typically have the disadvantage that the quality of the solutions decreases enormously with increasing temporal step sizes, which results from the numerical stiffness of the underlying partial differential equations. To prevent that, we apply a differential Thomas decomposition and a Lie symmetry analysis to derive explicit analytical solutions to specific parts of the Kirchhoff system. These solutions are general and depend on arbitrary functions, which we set up according to the numerical solution of the remaining parts. In contrast to a purely numerical handling, this reduces the numerical solution space and prevents the system from becoming unstable. The differential Kirchhoff equation describes the dynamic equilibrium of one-dimensional continua, i.e. slender structures like fibers. We evaluate the advantage of our method by simulating different scenarios, relevant in visual computing.

[1] King Abdullah University of Science and Technology
Thuwal, Saudi Arabia
dmitry.lyakhov@kaust.edu.sa

Applications of Computer Algebra – ACA2018
Santiago de Compostela, June 18–22, 2018

A symbolic-numeric method to determine symmetry of approximate differential equations

Zahra Mohammadi[1], **Greg Reid**[1]

We extended a critical point method based on penalty function introduced by Reid and Wu [1] to determining symmetry properties of general class of Differential Algebraic Equations DAE. This method interplay between geometric involutive form and numerical algebraic geometry which is based on homotopy methods.
There has been considerable progress on exploiting exact symmetry of exact system of DAE which using powerful symbolic packages such as ***rifsimp*** to get the involutive form of the system. These methods are less suited in applications since the coordinate dependency on ordering the variables can lead to numerical instability, especially on approximate systems. Our method applies a combination of geometric involutive form and critical point methods on the symmetry defining equations.
This work is sequel to [3] in which Numerical Linear Algebra is used to obtain an involutive of the system. In this work, we obtain useful information related all components of the involutive form of the DAE system and symmetry properties, by finding a witness points on each connected component. The approach exploits aspects of the termination of *Cartan- Kuranishi* theory of partial differential equations together with method of numerical algebraic geometry.

Keywords: Critical point, Numerical Algebraic Geometry, Symmetry.

References

[1] W.WU; C.CHANGBO AND G.REID, Penalty Function Based Critical Point Approach to Compute Real Witness Solution Points of Polynomial System *Computer Algebra in Scientific Computing*, 377-391, (CASC 2017).

[2] M. KURANISHI, On. E. Cartan's prolongation theorem of exterior differential systems. *Am. J. Math* **79**, 1-47 (1957).

[3] J.BONASIA; F.LEMAIRE; G.REID AND L.ZHI, Determination of approximate symmetries of differential equations. *Group Theory and Numerical Analysis*, 249-266 (2005).

[1]Applied Mathematics
Western University
Middlesex College,
1151 Richmond Street, London, Ontario
N6A 5B7 Canada
zmohamm5@uwo.ca
reid@uwo.ca

Applications of Computer Algebra – ACA2018
Santiago de Compostela, June 18–22, 2018

Challenges in Numerical Differential Algebra

Greg Reid[1], Zahra Mohammadi[1]

Much recent progress has been made in numerical polynomial algebra with the advent of homotopy-based methods and methods based on numerical linear algebra.

In this talk we review some of these developments, in the context of developing analogous methods for numerical differential algebra. A selecta of applications is given, including region dependent approximate symmetry, and determination of missing constraints in over and under-determined systems of partial differential equations with constraints. Geometric methods for such systems, together with stable methods form numerical linear algebra underly such approaches. Animations illustrating the application to approximate symmetries will be shown.

[1]Applied Mathematics
Western University
Middlesex College
1151 Richmond Street, London, Ontario
N6A 5B7 Canada
reid@uwo.ca
zmohamm5@uwo.ca