Applications of Computer Algebra – ACA 2019
Montréal, Canada | July 16-20, 2019
École de technologie supérieure

Surfaces and their Duals

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Recently I found among my many papers a one-page article about Dual Surfaces, written by Dr. Richard Morris, Liverpool University, which appeared in the Maths&Stats journal which was published for several years by the University of Birmingham. It must have been before 2000 because I couldn't find the article in the MSOR Maths&Stats archives:

https://journals.gre.ac.uk/index.php/msor/issue/archive and http://icse.xyz/mathstore/node/568-2.html

Dr. Morris wrote: *The dual of a surface is the set of planes tangent to the surface*.

The mapping works as follows:

Any plane $a\,x + b\,y + c\,z = d$ can be interpreted as a point (a,b,c,d) in $\mathbf{RP}^2$. Morris performs a projection of these points into $\mathbb{R}^3$ using the map

$$(a,\, b,\, c,\, d) \longrightarrow \left(\frac{a}{c}, \frac{b}{c}, \frac{d}{c}\right) \tag{1}$$

Some pictures in a low quality were presented. This was all. I will show how to perform this mapping using ***DERIVE*** and TI-Nspire CAS as well and present various surfaces together with their duals. Then I will vary the mapping followed by parameter curves and their duals. More questions came up, like "How does the dual of the dual look like?" This would be a nasty and boring calculation done by hand, but using a CAS … Finally, I don't raise the problem from the third dimension up to the fourth, but do it the reverse way: I make a step down and try to find the dual of a plane curve. Cusps appear, where do they come from? So, this short article of Dr. Morris kept me busy some time and brought a lot of pleasure and surprise into my mathematical life, which I would like to share with an auditorium.