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Proving and Disproving Subspaces with *Mathematica*

Aharon Naiman[1] [naiman@jct.ac.il]

[1] Department of Applied Mathematics, Jerusalem College of Technology, Jerusalem, Israel

Starting from the beginning of one's linear algebra education, one ventures into the area of vector spaces. After learning about the 10 axioms necessary for a vector space, the student delves into subspaces.

As a subspace is also a vector space, we know that we can go back to demonstrating that the 10 axioms are satisfied. However, there is a theorem that states that for a non-empty subset of a vector space, if the subset is closed under vector addition and scalar multiplication, then it is a subspace (and therefore also a vector space itself).

We present what tools are available to us in *Mathematica* [1], to assist in proving or disproving, in familiar mathematical notation, whether a subset is a subspace. First, we need to be able to model a vector space, where the vector subset resides. We demonstrate that we cannot do this, for all trivial vector spaces studied in an elementary course (e.g., function spaces).

Once we have the vector space, we present the necessary functions, together with their many options, to assist in proving that a subset:

1. is indeed a subspace—and *why*, i.e., the relationships of the results of the vector additions and scalar multiplications, or alternatively,
2. is *not* a subspace, and use additional forms of the available functions to demonstrate *intuitive* counterexamples.

For the students, the relationships of the results and the counterexamples are particularly important, in order to impart an instinctive understanding of the material. We further this understanding with some examples of demonstrating all 10 axioms to be fulfilled (or when some are not).

We develop proofs in both directions, using a few, different types of vector spaces, as well as various operations of vector addition and scalar multiplication.

Keywords
linear algebra; education; proving and disproving subspaces; *Mathematica*

References

Mathematica at www.wolfram.com/mathematica