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Boosting Rocket Performance without Calculus

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The stages of a two-stage rocket have initial masses m_1 and m_2 respectively and carry a payload of mass P. Both stages have equal structure factors e and equal relative exhaust speed c. The rocket mass, $m_1 + m_2$ is fixed and $\frac{P}{m_1+m_2} = b$.

According to multi-stage rocket's flight equation [2], the final speed of a two-stage rocket is

$$v = -c\log(1 - \frac{em_1}{m_1 + m_2 + P}) - c\log(1 - \frac{em_2}{m_2 + P}) \qquad (1)$$

.

Let $a = \frac{m_1}{m_2}$, (1) becomes

$$v = -c\log(1 - \frac{ea}{a + 1 + b(a + 1)}) - c\log(1 - \frac{e}{1 + b(a + 1)}) \qquad (2)$$

where $a > 0, b > 0, c > 0, 0 < e < 1$. We will maximize v with an appropriate choice of a.

The above rocket performance optimization problem is solved using calculus [1]. However, there is an alternative that requires only high school mathematics with the help of a Computer Algebra System (CAS). We reduce (2) successively to a new optimization problem where the target function is quadratic. The reduced problem is then solved analytically using high school level algebra (quadratic equation and inequality). This non-calculus approach places more emphasis on problem solving through mathematical thinking, as all symbolic calculations are carried out by the CAS [3]. It also makes a range of interesting problems readily tackled with minimum mathematical prerequisites.

Keywords
Optimization, Computer Algebra, High School Mathematics

References

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