

Intuition vs reality: automated exploration of envelopes with networking of technologies

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Envelopes of parameterized families of plane curves, of space curves, of surfaces, are an important topic both because of the mathematics involved and because of their applications (e.g. the determination of safety zones around sprinklers, robotic plants, Luna Park attractions, etc.). A drawback of this domain is the small number of its theorems, and the need to study numerous special cases [4].

The usage of technology revived the study of envelopes (e.g., see [1,2,3]). A Dynamic Geometry System (DGS) provides an environment for automated exploration and discovery, Nevertheless, the commands may not work in certain situations (such as non polynomial data or higher degree polynomials).

In such situations, it may be useful to transfer the data (the equations) to a Computer Algebra System, with which analytic solutions will be computed. The output may be transferred back to the DGS.

Let a parameterized family of plane curves \mathcal{C}_t be given by the equation $F(x, y, t) = 0$. If an envelope exists, it is given by the solution of the system of equations

$$\begin{cases} F(x, y, t) = 0 \\ \frac{\partial F}{\partial t} F(x, y, t) = 0 \end{cases}$$

In a polynomial setting, the **solve** command of the CAS uses algorithms from the theory of Gröbner bases [4]. In various situations, it is possible to transform the given data into polynomial form. Then the CAS provides a parametric presentation of the envelope (which can be described as the disjoint union of several components). These equations are copied into the DGS for the final graphical presentation (e.g. using the **Curve** command of GeoGebra).

In this talk:

1. We show how to use this "networking" of technologies is used;
2. We analyze the differences between the animations provided by the CAS and the interactive exploration enabled by the DGS, and how to have profit of these differences.

3. We analyze the possible contradiction between the first intuition and the actual output, in particular with regards to the issue of safety zones evoked above. In Figure 1), we show a family of circles centred on an astroid. The envelope of the family is different from the hull enclosing all the circles in the family.

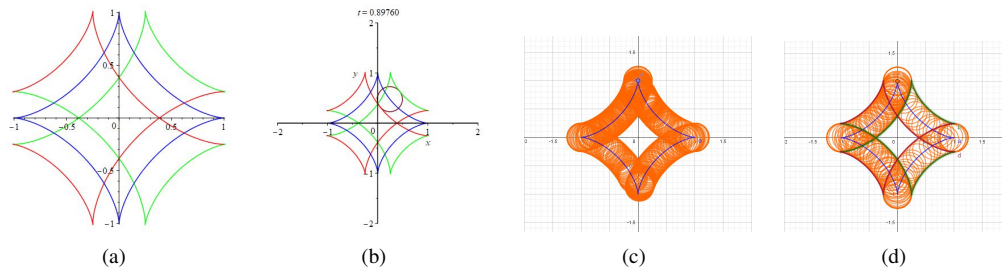


Figure 1: Envelope vs safety hull

Figure 1(a) shows the astroid and the envelope of a family of circles centred on it, with radius $1/4$, after computations with Maple. Figure 1(b) is a snapshot of an animation obtained with Maple, Figure 1(c) and Figure 1(d) show the output of a mouse driven experimentation with GeoGebra. Finally, Figure 1(d) shows the output after networking with the technologies. It emphasises the difference between the animation and the interactive exploration.

Keywords

Automated exploration, Envelopes, Networking

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