Applications of Computer Algebra – ACA 2021
Virtual. Online | July 23-27, 2021
Session on *"Computer Algebra in Education"*

CAS Tools for teaching function discontinuities

David G. Zeitoun[1]**,** ed.technologfie@gmail.com]

[1] Department of Mathematics, Orot College of Education, Elkana, D.N. Hare Efraim 44148, Israel

Proving that a function is continuous at a given point using the epsilon-delta definition is a difficult task for the student. Proving that the function is not continuous at a given point using the negation of the delta –epsilon definition is even more difficult.

In this work, we present new tools and applets from a computer algebra system (CAS) to enhance understanding of function discontinuities. The CAS assists the teacher at three levels (See [1];[2]):

- enouncing the basic definition of continuity/discontinuity;
- proving properties;
- helping to solve exercises.

We analyse the usage of the CAS (Geogebra) for the understanding of the definitions of discontinuity and various properties and present new CAS tools, commands and templates, such as a *control rectangle* for the visualisation of the definition of the continuity and discontinuity ([3]). The animations based on the control rectangle help to understand the delta-epsilon definition. We recall that Heine's theorem states that given any sequences (x_n) converging to a given point x_0, if the sequence $f(x_n)$ converges to $f(x_0)$ then f is continuous at x_0 .

Heine's theorem is hard to use in a proof of limit continuity at a point because we need to check the limit for any sequence. However, this definition of the limit permits to check if the function f(x) is not continuous at $x = x_0$. If two convergent sequences x_n and y_n both convergent to $x = x_0$, then if $lim(f(x_n)$ is different from $lim(y_n)$ $f(x)$ is not continuous at $x = x_0$. The Heine theorem allows to prove easily that a function is discontinuous at a point.

We address the following issues:

1. The δ ϵ - definition is difficult to understand intuitively and it is far from the intuitive understanding of the continuity and the discontinuity.

2. Moreover the $\delta \ \ \epsilon$ definition involves inequalities and requires algebraic knowledge in order to solve exercises. Therefore, the proofs of limit, continuity at a point is difficult for the student.

3. The definition of discontinuity may be taken from the $\delta \ \ \epsilon$ - definition. It require to find a value of ϵ such that for any value of δ ,$|x - x_0| < \delta$ and $|f(x) - f(x_0)| > \epsilon$. This definition is the negation of the $\delta \ \ \epsilon$ - definition of the continuity. So, the first difficulty for the student of this definition is to formulation the negation sentence of the continuity definition.

Keywords
Function discontinuity, CAS tools

References

[1] GIRALDO, V. AND CARVALHO, L.M. , *Local Straightness and Theoretical-Computational Conflicts: Computational Tools on the Development of the Concept Image of Derivative and Limit*, Proceedings of CERME 3, available: `http://www.dm.unipi.it/~didattica/CERME3/proceedings/Groups/TG9/TG9_Giraldo_cerme3.pdf`

[2] DUVAL, R., *The cognitive analysis of problems of comprehension in the learning of mathematics*, Educational studies in mathematics 61, 103-131. `https://doi.org/10.1007/s10649-006-0400-zl`

[3] ZEITOUN D.G,, DANA-PICARD, TH., In *Accurate visualization of graphs of functions of two real variables, International Journal of Computational and Mathematical Sciences 4(1), 1–11* . .