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# From hidden invariants to multiple solutions using computer algebra tools: two activities for pre-service teachers

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It is well known that the study of geometry involves a number of difficulties, one of which is the sharp switch from the intuitive exploration of the properties of geometrical objects to the necessity of rigorous proofs in the more advanced stages of learning geometry. To smooth this transition, modern curricula suggest involving learners in open-type activities through which they can explore various geometrical objects and the properties that arise in series of different geometrical situations.

Pre-service teachers do understand, at least in theory, the importance of both open-type and computer-assisted learning; a good portion of them are enthusiastic to take part in such a process. However, their initial trials often lead to early frustration as already during the initial steps of their inquiry search for the solution they encounter difficulty ("I have no idea how to get started!").

To overcome this difficulty, we have designed a set of open-ended activities, each of which begins with a very simple situation and then develops further to explore some profound - and often surprising - invariant properties. Here, we present two examples of such activities - portioning polygons and constructing Diophantine-type polygons - with a focus on how using computer algebra tools (GeoGebra in our trial) influences both students' reasoning and the results they achieve. The first activity, "From two to $n$, from one to infinity," deals with dividing a regular $n$-gon into $k$ congruent pieces (or at least into $k$ pieces with the same area). The origin/starting point is the partition of a regular $n$-sided polygon into $n$ equal triangular pieces. Further exploration will lead to numerous additional solutions, depending on the value of $k$ for the given regular polygon, and may even lead to a procedure for how to divide any arbitrary combination of $n$ and $k$ [1]. In any case, the discovery of area invariance through the use of GeoGebra provides the students breakthrough understanding of the infinite ways of division for every value of $k$.

The second activity described, "From nine to one, from one to infinity," is a three-step puzzle. The first step asks for the construction of polygons with an integer area from 12 whole toothpicks [2]. The immediately obvious area of 9 units (the square) can be easily decreased to 5 units even in the set of rectilinear polygons [3], and some students are able to discover

some polygons with even less area by using "manual" manipulations. However, to arrive at a large number of novel solutions, they typically need to explore perimeter invariance for the constructions in a dynamic environment, whereby they discover that the number of possible solutions is surprisingly huge - although finite. In the third part, the combination of two invariants pave the way to construct an infinite set of Diophantine - and even equilateral (!) - polygonal solutions.

**Keywords**
polygon divisions, Diophantine polygon construction, invariants, student reasoning

**References**

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