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Computer Algebra Systems – powerful tools for creating teaching-learning resources in undergraduate mathematics

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Ideas, approaches and tools for enhancement of undergraduate engineering mathematics are considered. A long lasting experiment with constant improvements in development and implementation of basic elements of holistic education - through development of the student's full potential, has proved a synergy effect of these activities. Some of them are represented. Computer Algebra Systems (CASs) are considered as a means to improve the overall effect of the quality of teaching-learning resources (TLR) on the student's learning path (trajectory). This quality is looked to with the hope of creating effective learning.

Two of the leading sentences for us are: "Knowledge can help you move from a point A to a point B, imagination can bring you from A to any other point" (A. Einstein) and "An individual's incorrect thoughts are due to insufficient development of his/her ability to distinguish" (Paramahansa Yogananda).

What we were trying to do is keeping the focus on the student/learner and the learning. We started with using different colors for different content and learning outcomes, e.g. new terms and definitions are highlighted in one color, important statements and sentences in another ([1, 2, 3, 4]) . In addition, different symbols enable a clear presentation of the content and make TLR easy to read/follow, e.g. a special symbol for pointing out that one often overlooks, ignores or wrongly understands or interprets.

Tips and rules are used to make it easier to work through the examples and exercises; structuring points and orientation aids are provided; the summaries are highlighted in color; important formulas and results are marked; model examples are appropriately placed in the text. Thoroughly calculated examples, tasks with solutions, illustrations and visualizations are included.

The next figures (figure 1 and figure 2) show an application of CAS *Derive* :

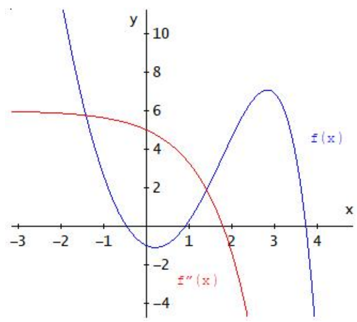

Figure 1: Graphical illustration of geometrical prototype of a sufficient condition for the convergence of an iterative method for solving nonlinear equations

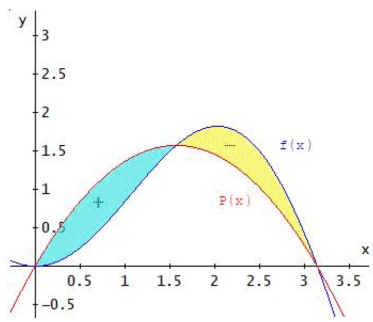

Figure 2: Graphical illustration of geometrical prototype of a rule for numerical integration and the error of the approximation value

The impact of colors on correct calculation of partial derivatives by students are illustrated by the next two equalities. And the correct rearrangement of terms in an ordinary differential equation is illustrated by the third one.

$$\Big(x^2\, y \sin x\Big)_x = \Big|\, y \text{ is treated as a constant } \Big| = y\, \underbrace{(2x\, \sin x + x^2\, \cos x)}_{\text{Product rule}}$$

$$\Big(\tan(x-y)\Big)_y = \underbrace{\frac{1}{\cos^2(x-y)}\,(x-y)_y}_{\text{Chain rule}} = \frac{1}{\cos^2(x-y)}\,(0-1) = -\frac{1}{\cos^2(x-y)}$$

$$\frac{\cos x}{1+y^2}\,\frac{dy}{dx} = \sin(x) \Rightarrow \frac{dy}{1+y^2} = \frac{\sin(x)}{\cos(x)}\,dx \Rightarrow \int \frac{1}{1+y^2}\,dy = \int \frac{\sin(x)}{\cos(x)}\,dx$$

CASs can be used for creating non-trivial questions to check the deepness of students' knowledge and to help them master the competence "reflection". For instance, we ask them to "read"/explain the lines #38 and #39 of figure 3. The student has to develop the ability for controlling the results (critical thinking), i.e. to built up the competence "reflection". If so,

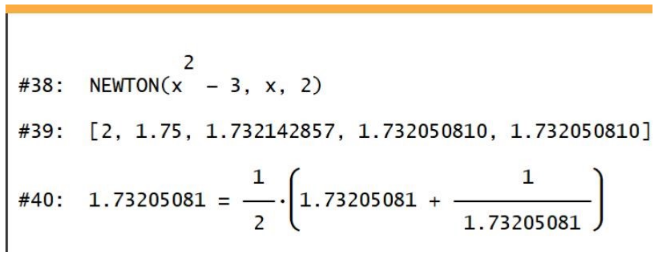

Figure 3: Newton's method with *Derive*

he/she can guess the relationship between the terms of the sequence in #39 and #40 of figure 3. For example, it has to be clear to him/her that there exists the relationship between the third and fourth number in #39 of figure 3.

And it has to be related to the third iteration obtained by Newton's method (abstract thinking):

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{3}{x_n}\right), n = 0, 1, 2.$$

In general, one needs to "read" symbolic, numerical, and graphical results and interpret them correctly. And to know that the components in the chain Knowledge-Skills-Control (Reflection) are interrelated. ("One can see as much as one knows.")

About Microlearning ([5]). Microlearning is a skill-based approach to learning that delivers information in small, highly focused chunks. A microlearning module (= a learning unit) is "as long as necessary and as short as possible". Learners tend to engage with microlearning more often, which increases learning retention. Microlearning is a strategy where independent learning units work for a single purpose and are part of the total learning picture.

Illustration of microlearning module and step-by-step (structured) approach to the solution. Solve the following ordinary differential equation of order one : $y' - y\tan(x) = \exp(\sin(x))$.
Solution.

- Step 0. Recognition of the type of the equation
- Step 1. Extraction of necessary information
- Step 2. Writing the formula for the general solution of a linear equation
- Step 3. Determination of the integrating factor
- Step 4. Replacement of the corresponding functions into the formula in Step 2
- Step 5. Solution of the integral in the right-hand side
- Step 6. Final answer (using Step 4 and Step 5): the general solution.

To communicate and collaborate with peers and engage on educational tasks students are provided questions for self-preparation; the opportunities of CASs are used for setting the questions up. The represented ideas and approaches for creating TLR can be used for blended or hybrid learning considered as a learning approach that combines traditional/conventional teaching-learning-assessment (TLA) process and remote learning activities. Purposeful and appropriate TLR could be effective in bridging the gap among remote and conventional learning and so to contribute to improve the hybrid learning. During the TLA process students develop learning abilities and habits, as well as educational values that are helpful for the real life and, above all, for their work. "Future of Work Is Nothing Without Consideration For The Future of Learning" ([6]). Future of learning solutions require the components of the triad Teaching-Learning-Assessment to become interdependent, not stand alone.

Keywords
Undergraduate mathematics, teaching-learning-resources, CASs, micro-learning.

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