Lectures on

Trace ideals in commutative algebra and combinatorics

by Jürgen Herzog [1]

Lecture 1: On the set of trace ideals of a Noetherian ring

We introduce trace ideals of modules and describe their basic properties. We classify the one-dimensional analytically irreducible local Gorenstein rings which admit only a finite number of trace ideals. This is joint work with Masoomeh Rahimbeigi.

Lecture 2: Nearly Gorenstein rings

Nearly Gorenstein rings were introduced by Takayuki Hibi, Dumitru Stamate and myself. They are the local Cohen–Macaulay rings with the property that the trace of the canonical module contains the maximal ideal. The canonical trace for tensor products and Segre products of Gorenstein algebras, as well as of (squarefree) Veronese subalgebras will be considered. The results are used to classify the nearly Gorenstein Hibi rings. Also the relationship between nearly Gorenstein and almost Gorenstein will be discussed. This is a report on joint work with Hibi and Stamate.

Lecture 3: Measuring the non-Gorenstein locus of Hibi rings and normal affine semigroup rings

In this lecture I report on joint work with Janet Page and Fatemeh Mohammadi in which we describe the non-Gorenstein locus of Hibi rings and of normal affine semigroup rings. The trace of the canonical module of a Segre product of algebras which are not necessarily Gorenstein will be considered, and the results will be applied to compute the non-Gorenstein locus of toric rings. Several necessary and sufficient conditions will be given for Hibi rings and normal semigroup rings to be Gorenstein on the punctured spectrum.

Lecture 4: The colength of the canonical trace and far-flung Gorenstein rings

For a numerical semigroup ring H, the colength of the canonical trace is compared with $g(H) - n(H)$, where $g(H)$ is the number of gaps of H and $n(H)$ is the number of non-gaps of H. One-dimensional Cohen-Macaulay rings whose canonical trace is as small as possible are those whose colength is as large as possible. Such rings are called far-flung Gorenstein rings. Far-flung Gorenstein rings will be considered especially for numerical semigroup rings. It is shown that the solution of the Rohrbach problem in additive number theory provides

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an upper bound for the multiplicity of far-flung Gorenstein numerical semigroup rings. This lecture covers joint work with Dumitru Stamate and Shinya Kumashiro.

Lecture 5: Rings of Teter type

A 0-dimensional local ring $(R, \mathfrak{m}_R)$ is called a Teter ring, if there exists a local Gorenstein ring $(G, \mathfrak{m}_G)$ such that $R \cong G/(0 : \mathfrak{m}_G)$. This class of rings has been introduced 1974 by William Teter. It has been shown by Huneke and others that a local ring R is a Teter ring if and only if there exists an epimorphism $\varphi\colon \omega_R \to \mathfrak{m}_R$. Here ω_R denotes the canonical module of R. The ring is called of Teter type, if there exists a surjective homomorphism from ω onto the trace of ω_R. Various classes of algebras of Teter type will be considered. This is joint work with Oleksandra Gasanova, Takayuki Hibi and Somayeh Moradi.