Near–MDS codes and caps

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Let q be a power of a prime and $\mathrm{PG}(k-1,q)$ the projective space of dimension $k-1$ over $\mathbb{F}_q$. We call *n-cap* a point set of size n such that no three of them are collinear; it is complete if it is not contained in any $(n+1)$-cap. If we take the matrix whose columns are the representative of the points of an n-cap, we get the parity-check matrix of a linear code over $\mathbb{F}_q$. Moreover, if $n > k$, complete n-caps of $\mathrm{PG}(k-1,q)$ are essentially equivalent to non-extendable linear $[n, n-k, 4]_q$ codes with covering radius $\rho = 2$.

For any $[n, k, d]_q$ linear code, the *Singleton defect* is $D := n - k + 1 - d$. We call *near-MDS* a code such that both itself and its dual have $D = 1$ and this is equivalent to say that the columns of a generator matrix form a set of points in $\mathrm{PG}(k-1,q)$, $k \geq 3$ (called NMDS-set) with the following three properties: every $k-1$ points generate a hyperplane, there are k points belonging to the same hyperplane and every $k+1$ points generate the whole $\mathrm{PG}(k-1,q)$. An NMDS-set is *complete* if it is maximal with respect to inclusion.

In this talk, based on the paper [1], we will examine NMDS-sets of dimension 4 and caps in $\mathrm{PG}(4,q)$. In particular we will see: a class of NMDS-sets of $\mathrm{PG}(3,q)$, $q = 2^{2h+1}$, $h \geq 1$, obtained intersecting an elliptic quadric and a Suzuki–Tits ovoid of $W(3,q)$ (size: $q + \sqrt{2q} + 1$), two classes of complete caps of $\mathrm{PG}(4,q)$, derived by the previous result (size: $2q^2 - q \pm \sqrt{2q} + 2$) and the possible sizes of an NMDS-set containing a twisted cubic of $\mathrm{PG}(3,q)$.

References

[1] Michela Ceria, Antonio Cossidente, Giuseppe Marino, Francesco Pavese, *On near–MDS codes*, arXiv:2106.03402 [math.co]