Two-point AG codes from the Beelen-Montanucci maximal curve

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Algebraic geometry codes (AG codes) are a family of error-correcting codes introduced by Goppa in the '80s (see [5] and [6]) and constructed using algebraic curves defined over a finite field. A general lower bound for the minimum distance of an AG code is given by the well-known Goppa bound, so that for a code $[n, k, d]$ whose underlying algebraic curve has genus g, the inequality $d \geq n - k + 1 - g$ holds, hence the minimum distance can be designed.

Let $\mathbb{F}_q$ be the finite field with q elements and $\mathcal{X}$ be an algebraic curve defined over $\mathbb{F}_q$ and of genus g. $\mathcal{X}$ is said to be maximal if it attains the Hasse-Weil bound, which means it has the largest number of rational points with respect to its genus. For this reason, maximal curves are suitable candidates for the construction of AG codes with good parameters.

In this talk, I will present some results (see [7]) on duals of two-point AG codes coming from the Beelen-Montanucci maximal curve, introduced in [3]. In particular, we used the order bound ([2]) to compute a lower bound on the minimum distance that improves the Goppa bound. Our results rely on the study of a certain two-point Weierstrass semigroup (see [4]) on the Beelen-Montanucci curve, which we managed to determine completely. Using these methods, we discovered AG codes with better parameters with respect to comparable two-point codes (investigated in [1]) from the Garcia-Gneri-Stichtenoth (GGS) curve.

References

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