On the relationship between irreducible cyclic codes, finite projective planes and non-weakly regular bent functions

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It is known that there is a one-to-one correspondence between irreducible cyclic codes over finite fields and multiplicative subgroups of finite fields. Namely, q being a prime power, and choosing a multiplicative subgroup of order n of a finite fields of order q^m as a defining set, one can obtain an irreducible cyclic $[n, m_0]$ code over $\mathbb{F}_q$ based on the generic construction method introduced by C. Ding, where m_0 divides m [3]. The main problem is to evaluate the weight distribution of these codes, which depends on the Gaussian periods of the cyclotomic classes of order N in $\mathbb{F}_{q^m}$, where $q^m - 1 = nN$. In [6], we observed that two disjoint subsets, $B_+(f)$ and $B_-(f)$, of the finite fields of order 3^6 obtained by partitioning the field with respect to the signs of the Walsh spectrum of a sporadic example of ternary non-weakly regular bent function f could be written as a union of certain cosets of the cyclotomic classes of order 13 in F_{3^6}. Furthermore, we observe that irreducible cyclic code obtained by using the multiplicative subgroup of order 56 in $\mathbb{F}_{3^6}$ as a defining set is three-weight. As a union of certain cosets of this defining set in $\mathbb{F}_{3^6}$, $B_+(f)$ and $B_-(f)$ give rise to fusion schemes of class 2 (strongly regular graphs), and so two-weight projective linear codes. In this talk, after reviewing the general features, I will survey our further observations on the relationship between those structures and finite projective planes.

References

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