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Using CAS in the classroom: personal thoughts (Part III)

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In memory of Eugenio Roanes-Lozano

At ETS, CAS technology is mandatory since 1999 and TI-Nspire technology is used campus wide since 2011 (both calculator and software). The talk will be the third of a series about how technology has changed the way we teach mathematics to future engineers. First, let us recall what we did in the last two ACA conferences.

For the online 2021 Athens conference, a third degree polynomial equation was chosen. Newton's method was applied and animated in order to find the roots. Cardano's formulae were deduced using CAS manipulations. We showed why trigonometric substitutions are a better choice when all roots are real.

For the 2022 hybrid Istanbul conference, complex analysis was chosen. We visualized the complex roots of a polynomial using 2D and 3D plots using TI-Nspire CAS. We showed how Laurent series, residue integration techniques and numerical line integrals can be combined to verify some answers. And how the Maple built-in Rieman Zeta function allows us to observe some non trivial zeros of $\zeta(s)$.

For this in person 2023 Warsaw conference, ODEs and real analysis are chosen. The examples listed below are interesting for a student to explore when a CAS handheld is available during the classroom. Using some popular computer algebra systems, the examples will be performed live during the talk.

- Heavy computations are often required in ODEs application problems, so using a CAS has always been natural. But theoretical results (namely the existence and uniqueness of solutions) can benefit from CAS computations and graphic facilities. It is quite interesting to look at the answer provided by popular CAS when solving

  $$\frac{dy}{dx} = \frac{4y}{x^2-9}, \quad y(a) = b\,.$$

  For some initial values, hidden complex numbers can appear on the screen due to cubic roots. This is a good opportunity to recall the domain of a solution. The textbook [1] is famous for this : the authors took special care of linearity and effects of parameters on solutions instead of listing many tricks for solving different ODEs.

- Engineering students rarely use mathematical analysis. Pointwise convergence of series of functions is not an important part of their curriculum. At ETS, the differential equations course has been updated recently (see [2] where you can download my colleague Gilles Picard's *Volume* 2). Taylor series method for solving ODEs along with numerical methods are used for linear variable coefficients second order ODEs. Example : without technology, one can find a series solution $y(x) = \sum_{n=0}^{\infty} a_n\, x^n$ for the problem $(x^2+1)\, y'' - 3y' + y = 0, y(0) = 1, y'(0) = -1$. But when the following recurrence formula will be found, we will need a machine to compute the coefficients :

$$a_n = \frac{3(n-1)\, a_{n-1} - (n^2 - 5n + 7)\, a_{n-2}}{n(n-1)},\ n \geq 2.$$

  Then if you want to compute **with accuracy** the value of the solution at some point x_0 of the interval of convergence, you need to use a partial sum of the form

$$so(p) = \sum_{n=0}^{p} a_n\, x_0^n.$$

  Finally, a numerical first-order system ODE solver can validate the answer.

- Equation solving can benefit from CAS graphical capabilities. A simple example as solving $x^x = a$ for different values of the parameter a can easily force the students to use calculus and get to know the LambertW function!

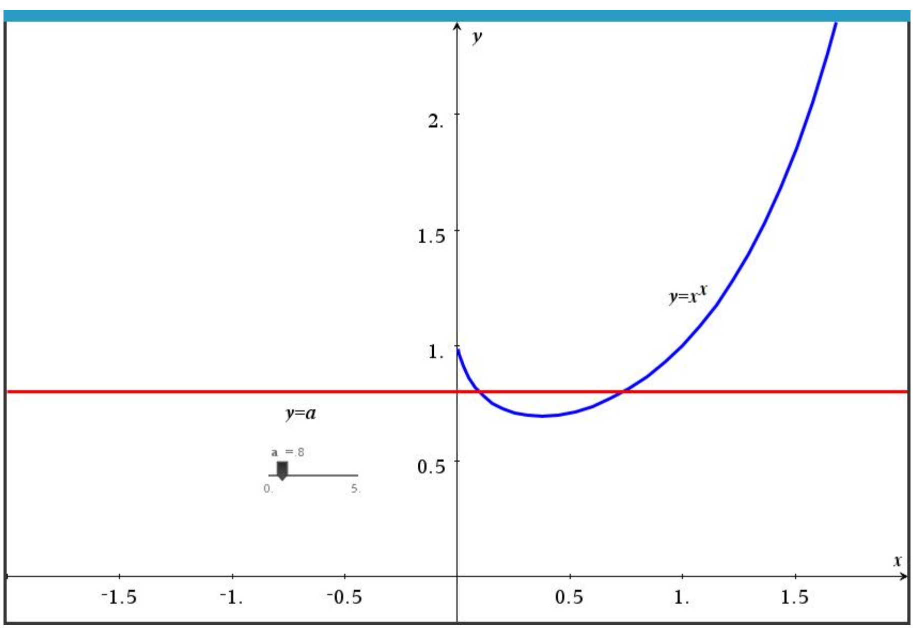

Figure 1: Horizontal line intersecting the curve x^x

References

[1] Kostelich, Eric J.; Armbruster, Dieter, *Introductory Differential Equations. From Linearity to Chaos*. Addison Wesley, 1997.

[2] https://luciole.ca/gilles/mat265/.

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