Applications of Computer Algebra – ACA 2023
July 17-21, 2023
Session on *"Computer Algebra in Education"*

Surfaces and their Duals

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In memory of Eugenio Roanes-Lozano

Recently, I found among my many papers a one-page article about *Dual Surfaces*, written by Dr. Richard Morris, Liverpool University, which appeared in the Maths&Stats journal, which was published for several years by the University of Birmingham. It must have been before 2000 because I could not find the article in the MSOR Maths&Stats archives : see [1] and [2]. Dr. Morris wrote: *The dual of a surface is the set of planes tangent to the surface*. The mapping works as follows. Any plane $a\,x + b\,y + c\,z = d$ can be interpreted as a point (a, b, c, d) in RP^3. Morris performs a projection of these points into $\mathbb{R}^3$ using the map

$$(a, b, c, d) \to \left(\frac{a}{c}, \frac{b}{c}, \frac{d}{c}\right).$$

Some pictures in a low quality were included. This was all!

I will show how to perform this mapping using *DERIVE* and TI-Nspire CAS as well and present various surfaces together with their duals. Cusps appear and we can ask where they are coming from? Then I will vary the mapping followed by parameter curves and their duals. More questions come up, like *How does the dual of a dual look like?* This would be a nasty and boring calculation done by hand, but using a CAS this becomes possible. Finally, I don't raise the problem in a higher dimension – as generalisation usually works – but do it the reverse way: I make a step down and try to find the dual of a plane curve. Again cusps appear. Where do they come from? Preparing this presentation I did some internet research – and I was lucky enough to find in [3] another much more extended paper published by Morris also in 2002 – which provided more insight for the properties of the duals. So, this short article of Dr. Morris from 2002 kept me busy some time and brought a lot of pleasure and surprise into my mathematical life which I would like to share with an audience. I'll show among others a dual with cuspidal edges (Fig 2), how they can be derived and from where they are coming from in the base surface (Fig 1). The references could be accomplished with [4].

Keywords
Dual surfaxes, Mappings, Gauss Curvature, Graphic Representations

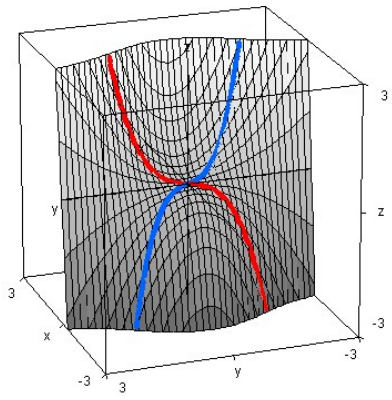

Figure 1: Base surface

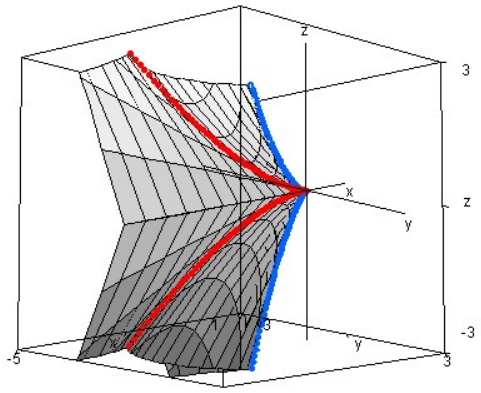

Figure 2: Dual with cuspidal edges

References

[1] https://journals.gre.ac.uk/index.php/msor/issue/archive

[2] https://icse.xyz/mathstore/node/568-2.html

[3] https://www.singsurf.org/papers/dual/dual.pdf

[4] Alfred Gray, *Differentialgeometrie*, Spectrum 1994.