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Automated computation of geometric Loci in Mathematics Education

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In memoriam of Eugenio Roanes-Lozano.

The computation of geometric loci is an important topic, both at High-School level and at undergraduate level. This topic has been explored for a long time [1,2]. Nevertheless, the goals and the methods which can be utilized with todays' tools are different. At undergraduate level, the dialog between Geometry and Algebra is fruitful [3, 5], but it is less relevant for High-School. For example, section 4.2 in [3] emphasizes the process of obtaining locus visualization (image, equation), and considers sophisticated examples, far from the contents of the pre-university education in most countries. Moreover, it does not develop the two steps that we consider more relevant when dealing with secondary education.

These steps will be the main points of attention in our contribution here:

- conjecturing the loci structure —and not only its equation or plot—
- and verifying the soundness of the conjecture—with automated reasoning tools.

In our contribution, we will argue how both tasks should be, in the educational context, the more relevant ones that students of today (or tomorrow) would have to learn. We exemplify these ideas with two examples:

1. Ptolemy's theorem;
2. a very simple, yet not obvious, locus, namely that of the vertex C of a triangle ABC, such that the medians from A to the midpoint of AC and from B to the midpoint of BC are perpendicular (Figure 1).

As an example of our disquisitions, let us mention that, in the second example, one immediate output of the exploration can be an equation for the desired geometric locus, but this equation

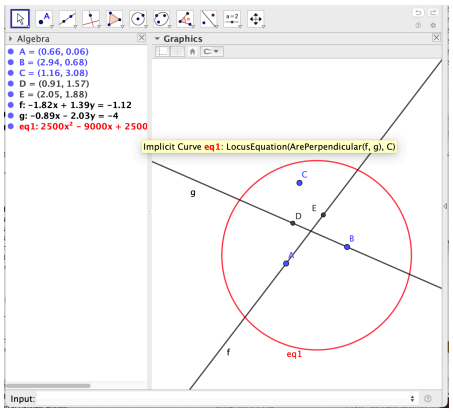

Figure 1: Automated determination of a locus

is only computed numerically, and it might be of little help for a High-School student searching for some geometric features (center, radius) of this locus. Moreover, the plotted circle is just a "shape", not a geometric object recognized by the software, and standard GeoGebra commands such as "Prove" (also available as a button) cannot apply to verify conjectures over this plot. Thus, students have to explore and confirm their conjectures with the use of more advanced tools offered by GeoGebra-Discovery [4] that provide an exact answer, and enable to determine "what is really" the desired locus, after implementing a "true' geometric construction of the circle, using plane transformations which have automated implementations in the software.

Keywords
Geometric Locus, GeoGebra-Discovery, Automated Proof

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