Applications of Computer Algebra – ACA 2023
July 17-21, 2023
Session on *"Computer Algebra in Education"*

# Rubi gems

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## 1 Abstract

The Rule-Based Integration project (Rubi) has been under continuous development since 2008. The current publicly available release is version 4.16.1.0 available at [1]. Active development of Rubi's database of rules has continued unabated since that version. This report selects a few of the recent additions to Rubi, and describes some of the processes that help direct the research into new rules.

We recall that while the primary aim of Rubi is to evaluate indefinite integrals using a rule-based paradigm, an important aspect of this aim is that Rubi is not content to return *just any* expression that is a valid integral, but aims to return *the best* or *optimal* integral expression.

### 1.1 The best integral

How do we decide which of these integrals is better?

$$2\sqrt{2}\int \frac{t^2}{1+t^4}\,dt = \arctan\frac{-1+t^2}{\sqrt{2}\,t} - \operatorname{arctanh}\frac{\sqrt{2}\,t}{1+t^2}\,, \tag{1}$$

$$= \arctan(\sqrt{2}\,t+1) + \arctan(\sqrt{2}\,t-1) - \operatorname{arctanh}\frac{\sqrt{2}\,t}{1+t^2}\,. \tag{2}$$

We shall discuss this question.

### 1.2 Some new trig integrals

One of Rubi's great strengths is special cases. The integral below is one special case of a general class. Using the rule for the general class results in a very long answer. In new Rubi,

the special case is recognized and returned.

$$\int \frac{\sin(100x)+\sin(99x)}{\cos(100x)+\cos(99x)}\,dx = -\frac{2}{199}\ln\cos\frac{199x}{2}\ .$$

**Keywords**
Symbolic integration, rule-based systems, optimal expression analysis

**References**

[1] A. D. RICH, *Rubi (Rule-based Integrator)*,
`https://rulebasedintegration.org/about.html`