Applications of Computer Algebra – ACA 2023
July 17-21, 2023
Session on *"Computer Algebra in Education"*

GeoGebra Automated Reasoning Tools: why and how (to use them in the classroom)

Zoltán Kovács[1]***, Tomás Recio***[2]***, M. Pilar Vélez***[2] [pvelez@nebrija.es]

[1] The Private University College of Education of the Diocese of Linz, Linz, Austria
[2] Escuela Politécnica Superior, Universidad Antonio de Nebrija, Madrid, Spain

In memory of Eugenio Roanes-Lozano

Dynamic Geometry (DG) systems have become quite popular for teaching purposes. Although initially characterized by the ability to drag geometric constructions while keeping the relations (perpendicularity, parallelism, etc.) established by the user among the involved geometric objects, some of these environments evolved to integrate a computer algebra system, so they should be better described as "dynamic mathematics" software. This is the case of GeoGebra*, integrating the computer algebra system Giac. This has opened the possibility to translate algebraically the geometric relations between objects in a construction and, thus, to deal with computer algebra algorithms that allow the development of highly performing, mathematically rigorous (not just probabilistic or numerically approximate) Automatic Reasoning Tools (ART) on geometric statements. Thus, currently GeoGebra (and, for some advanced tools, the fork version GeoGebra Discovery†) already offers the user a rich variety of ART for tasks related to experimenting, discovering, and asserting:

- Automatically declaring the truth or failure of a given statement (`Prove` and `ProveDetails` commands),
- Automatically discovering how to modify a given figure so that a wrong statement becomes true (`LocusEquation` command, returning where to place some point on a construction so that a given property holds),
- Automatically discovering and returning a message with the properties holding among some selected pair of elements of the given figure (`Relation` command),
- Automatically discovering all statements (of a certain kind: lengths ratio, perpendicularity, etc.) holding true and involving a given element in a figure, selected/introduced by the user (`Discover`, `StepwiseDiscovery` command),

*http://www.geogebra.org
†https://github.com/kovzol/geogebra/releases

- Automatically discovering all statements of a certain kind involving "all" the elements of a given figure, see ‡.

Unfortunately, the algebraic geometry nature of the algorithms behind these tools does not allow providing readable arguments justifying their outputs. Yet, we think that the universal accessibility and portability of GeoGebra; its worldwide diffusion, with more than 100 million users all over the world – especially in the educational field – requires the analysis and design of new approaches to teaching proof at secondary education level and beyond, because teaching geometry to students that have at their disposal powerful ART which they can apply to deal with geometric problems, cannot be a mere repetition of the traditional curriculum: teachers should learn about this technology and should explore its application in the classroom

As it was already noticed by Howson and Wilson, too long ago, in the publication known as Kuwait report [1]:

> ". . .today's student may well be able to apply algebraic methods. . . The solution derived by applying a mechanical procedure may be less aesthetically satisfying than a geometrical one, but are there other objections to algebraic methods than that of aesthetics?"

As well as very recently in a paper by Hanna and Yan [2]:

> "It is perhaps too early for empirical studies of classroom experience using the enhancements to GeoGebra. In this respect the situation of GeoGebra is similar, but not identical, to that of proof technology in general. While it is reasonable to expect proof technology to foster students' proving abilities, and there is certainly supporting anecdotal evidence, its potential advantages have not yet been systematically assessed. . . Theorem provers do provide a guarantee, as we have seen, but in the shape of a fully formal proof that may be unintelligible. . . This state of affairs is a challenge for educators. . . They also have reason to believe, based on the anecdotal evidence, that this new proof technology could turn out to be of great benefit in the classroom. . . The key is to make a start, beginning with exploratory studies of the potential of these new tools at both the secondary and post-secondary levels."

Thus, the purpose of this talk is, firstly, to make a summary presentation of the above-mentioned automated reasoning functions in GeoGebra, through some illustrative examples [3], [4]. Then we will focus on the proposal of diverse open-ended tasks, inspired in recent experiences ([5], [6], [7]) that have been developed with different kinds (secondary education, undergraduate or initial teacher training) of students, regarding the use of automated reasoning techniques, showing how these tools can be used within the educational context, helping students to develop "augmented intelligence" skills by reasoning in collaboration with the computer.

Acknowledgements: Authors are partially supported by grant PID2020-113192GB-I00 (Mathematical Visualization: Foundations, Algorithms and Applications) from the Spanish MICINN.

‡ `https://autgeo.online/ag/automated-geometer.html?offline=1`

Keywords
Automated reasoning in geometry, GeoGebra, Math instruction, Augmented intelligence

References

[1] G. HOWSON; B. WILSON, *ICMI Study series: School mathematics in the 1990's*. Cambridge University Press, 1986.
[2] G. HANNA; X. YAN, Opening a discussion on teaching proof with automated theorem provers. *For the Learning of Mathematics* **41**(3), 42–46 (2021).
[3] Z. KOVÁCS; T. RECIO; P.R. RICHARD; S. VAN VAERENBERGH; M.P. VÉLEZ, Towards an Ecosystem for Computer-Supported Geometric Reasoning. *International Journal for Technology in Mathematics Education (IJTMT)* **53**(7), 1701–1710 (2022).
[4] Z. KOVÁCS; J. YU, Stepwise discovery of geometrical knowledge in GeoGebra. *Proceedings of The 8th Czech – Slovak Conference on Geometry and Graphics 2022*, M. Lavicka, M. Chodorová, L. Rýparová, D. Szarková (eds.) Plzeñ, 2022.
[5] T. RECIO; P.R. RICHARD; M.P. VÉLEZ, Designing tasks supported by GeoGebra Automated Reasoning Tools for the development of mathematical skills. *International Journal for Technology in Mathematics Education (IJTMT)* **26**(2), 81–89 (2019).
[6] J.M. FORTUNY; T. RECIO; P.R. RICHARD; E. ROANES-LOZANO, Análisis del discurso de los profesores en formación en un contexto de innovación pedagógica en geometría. *Annales de Didactique et de Sciences Cognitives* **26**, 195–220 (2021).
[7] A. HERRERO; T. RECIO; P. TOLMOS; M.P. VÉLEZ, From the Steam Engine to STEAM Education: An Experience with Pre- Service Mathematics Teachers. *Mathematics* **11**(2), 473 (2023).