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Using CAS in Mathematics Education with The Quadratic Curve Addition Method

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The author has proposed a geometric construction that adds the locus of a quadratic curve to a drawing with a ruler and compass. The author calls this method “the quadratic curve addition method." Its characteristic method is based on the fact that the student discerns the foci and directrix, the characteristics of a quadratic curve, from the given conditions. Therefore, the author has made it possible for KeTcindy and LaTeX to generate quadratic curves by telling the dynamic geometry software (Cinderella) the foci and directrix, which are the characteristics of the quadratic curves that they have detected. The circle’s center, found by the quadratic curve addition method, is the intersection of those trajectories and figures. The author has shown the effectiveness of this method using Apollonius’ problem [1].
In the following, the author presents a case study of the quadratic curve addition method, using the Japanese Sangaku as a subject to show a drawing of a quadratic curve. Concerning the locus of a quadratic curve and CAS, for example, when finding the radius of a circle, it is helpful to express the locus and conditions in terms of equations. In particular, equations representing quadratic curves are generally complex, and the author believes that obtaining solutions by substituting them into CAS, such as Wolfram Alpha, is effective in mathematics education.

The Educational Value of Drawing
Geometric construction is an activity with high educational value. Through such activities, as a result, it is expected to enhance students’ problem-solving abilities.
In connection with this study, recent dynamic geometry software has excellent features in GUI plotting and CUI plotting. Therefore, it is possible to transfer mathematical concepts to the software by converting them into scripts, which is expected to provide more opportunities to use mathematics better. If scripting becomes possible, it is conceivable to create a unit vector on the two sides that flank the angle and to construct the angle bisector as the sum of the two vectors. Another possible method is to express one vector of the inner center by the position vectors of the three vertices. The author’s mathematical utilization is the teaching method that can be realized by writing mathematics in scrips.

Problem: As shown in Fig.1, place the large circle O so that circles A, B, C, and D are tangent to the interior of the significant process. Let the diameters of the circles O and A be 16 cm, 4.8 cm, respectively. Note that the author quoted Sangaku’s problem as an example of a drawing using the quadratic curve addition method.

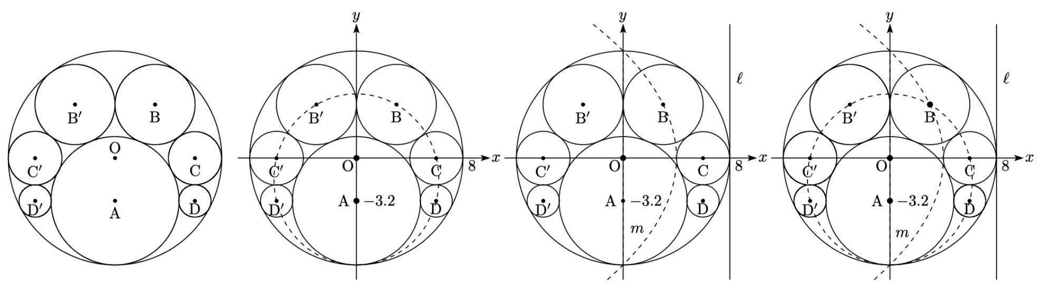

Fig.1 Fig.2 Fig.3 Fig.4

Q1: Find the center of circle B by the quadratic curve addition method.
A1: The center of circle B lies on the locus of an ellipse with point O and point A as foci. However, the sum of the distances from the two points O and A is the sum of the radii of the circles O and A. The center of circle B also lies on the locus of a parabola with O as the focal point and line ℓ as the directrix. □

Q2: Find the radius of the circle B.
A2: In Fig.2, the center of the circle B is on the ellipse.

$$\sqrt{x^2+y^2}+\sqrt{x^2+(y+3.2)^2}=12.8$$

And in Fig.3, the circle's center B lies on the parabola with point O as the focal point and ℓ as the directrix.

$$x=-\frac{1}{16}y^2+4$$

As shown in Fig.4, the point B is the intersection of an ellipse and a parabola.
We solve the system of equations; we obtain $(x,y)=(3,4),(0,-8)$.
Hence, the center of circle B is $(3,4)$.
Since $\mathrm{OB}=5$ cm and the radius of circle O is 8 cm, the radius of circle B is 3 cm. □

The author will present the quadratic curve addition method for some Sangaku problems in this talk. The author will also share the significance and value of the quadratic curve addition method in mathematics education and deepen the discussion with the participants.

Keywords
the quadratic curve addition method, Cinderella, KeTCindy, LaTeX, Wolfram Alpha, Sangaku

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References
[1] H. MAKISHITA, The problem of Apollonius which was solved by Geometric Construction and Loci of Quadratic Curves. *Transactions of mathematical education for Kosen and University* **25**, 65–86 (2019).