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Orthogonal trajectories to isoptics of ovals

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Let C be an oval (by which we mean a simple closed convex plane curve of class C^2 with positive curvature) and $\alpha \in (0, \pi)$. The set of points at which two support lines of C intersect at angle $\pi - \alpha$ is called an α-isoptic (or simply an isoptic) of C. Isoptics of plane curves are most often considered in the parametric form proposed in [1] and this parametrization seems to be the main tool in the study of isoptics and their generalizations, see for example [2], [5], [6], [7], [8]. Isoptics can be considered also in the nonparametric form, however, implicit equations are known only for a small class of curves, see for example [2],[3].

Our goal is to find orthogonal trajectories of isoptics, but not using the classical approach to this task, which uses implicit equations. We construct parametrizations of orthogonal trajectories to isoptics of ovals, using the solution of a specific Cauchy problem. To prove that the defined function is continuous, we use some version of l'Hôpital's rule for multivariable functions [4]. To illustrate the problem, we analytically determine orthogonal trajectories for a simple example of a circle isoptics, while for more complicated examples we provide and draw numerical solutions, created using the Mathematica program.

In addition to discussing the subject of my research, I'll also share some experiences regarding teaching differential geometry at the University.

Keywords
isoptic curve, support function, evolution, orthogonal trajectory

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