The Shimizu–Morioka System Has No Nontrivial Darboux Polynomials

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In 1980 Shimizu and Morioka [3] presented a simple three dimensional ordinary differential equation as a model to study the convection of turbulent flows (i.e. flows with high Rayleigh numbers). More recently, Huang et al. [2] studied the Darboux integrability of the system and showed that it has no nontrivial Darboux polynomial of total degree less than four. They further conjectured that the system has no nontrivial Darboux polynomial for any positive total degree. We prove that this is indeed the case leveraging our seminal work on using the concept of *generic polynomials* to systematically study the existence of Darboux polynomials [1].

This is a joint work with Maxime Bridoux (Inria, France).

References

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