Girth Analysis of Quantum Quasi-Cyclic LDPC Codes

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Quantum error-correcting codes (QECCs) are vital for safeguarding quantum information from the detrimental effects of decoherence and quantum noise. This makes them crucial in quantum computing and communication. While quantum computers have the potential to solve problems much faster than their classical counterparts [4], they are highly susceptible to errors. Addressing these errors is a major challenge, and QECCs have become a key strategy to protect quantum information. The concept of QECCs was initially introduced in foundational works by Calderbank, Shor, and Steane [2] and [5]. The Calderbank-Shor-Steane (CSS) framework has provided a cornerstone for much of the subsequent research in the field.

Quantum quasi-cyclic LDPC (QQC LDPC) codes, like CSS codes have good structure and popular channel coding schemes. We investigate the use of fully connected quasi-cyclic LDPC (QC-LDPC) codes to build QQC-LDPC codes. It is known (experimentally) that the girth, that is, the length of the shortest cycles of the bipartite graph of its parity-check matrix, influences the code performance.

We prove [1] that QC-LDPC codes with column weight $\mathcal{J}$ at least 3, used to construct QQC-LDPC codes have girth at most 6. We present an efficient and practical method to obtain QQC-LDPC codes from QC-LDPC codes with $g = 8$ and $\mathcal{J} = 2$. Then, we extend our method to construct codes with $\mathcal{J} = 2$ and $g = 12$, thus reaching the largest possible girth [3].

References

[1] F. Amirzade, D. Panario and M.-R Sadeghi, "Girth Analysis of Quantum Quasi-Cyclic LDPC Codes", *Problems of Information Transmission* 60, 71–89, 2024.

[2] A. R. Calderbank and P. W. Shor, "Good quantum error-correcting codes exist," *Phys. Rev. A* 54, 1098–1105, 1996.

[3] M. P. C. Fossorier, "Quasi-cyclic low-density parity-check codes from circulant permutation matrices," *IEEE Trans. Inf. Theory* 50, 1788–1793, 2004.

[4] P. W. Shor, "Scheme for reducing decoherence in quantum memory", *Phys. Rev. A* 52, 2493–2496, 1995.

[5] A. M Steane, "Simple quantum error-correcting codes," *Phys. Rev. A* 54, 4741–4751, 1996.