Universal polarization of sharp codes in the Leech lattice

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Given a spherical code $C \subset \mathbb{S}^{n-1}$ and a potential h, the discrete h-potential of C is given as $U_h(x, C) = \sum_{y \in C} h(x \cdot y)$. A spherical $\tau = 2k - 1$ or $\tau_{1/2}$-design (a τ-design with vanishing moments of order $\tau + 2$ and $\tau + 3$), that can be embedded in k or $k+1$ parallel hyperplanes is called *PULB*-optimal, i.e. attains a polarization universal lower bound below. For a PULB-optimal code C and very broad class of potentials the location of the global minima of $U_h(x, C)$ are universal and independent of h. Two PULB-optimal codes C and D are called PULB-optimal pair (C, D) if the universal minima of $U_h(x, C)$ are the points of D and vice versa, the universal minima of $U_h(x, D)$ are the points of C. We call a PULB-optimal pair maximal if D is the set of all universal minimal of $U_h(x, C)$ and vice versa. We shall show that some remarkable universally optimal codes embedded in the Leech lattice give rise to maximal PULB-pairs.

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