A direct solver for coupled systems of recurrence equations over $\Pi\Sigma^*$-fields

Jakob Obrovsky
Johannes Kepler University Linz, Austria

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In this talk I am going to present my work in cooperation with Carsten Schneider on a direct solver for coupled systems of linear higher-order recurrence equations whose coefficients are given in terms of nested sums and products.

One strategy for solving such systems is to first decouple the system [3] and [10] to obtain several scalar equations in only one of the unknowns. These scalar equations then can be solved using the algorithm in [2]. However, this strategy is rather inefficient if the dimension of the system is large [6]. For some cases, algorithms have been developed to avoid decoupling and solve the system in a direct way. These algorithms efficiently find hypergeometric and rational solutions for the rational difference field $K(x)$ with $\sigma(x) = x + 1$ and rational solutions of systems of q-recurrence equations [1], [5] and [8]. We generalized these methods to obtain an algorithm that operates over $\Pi\Sigma^*$-fields [7], incorporating ideas from [4] and [9]. That is, we can directly compute hypergeometric and rational solutions of coupled systems of recurrence equations over $\Pi\Sigma^*$-fields. Within $\Pi\Sigma^*$-fields it is possible to represent indefinitely nested sums and products, thus covering in particular a big class of D-finite sequences.

During the talk I will give a rough overview of the main components of the solver and present examples from the wide range of inputs for which the solver is applicable.

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