

Computer Algebra Systems in Secondary Mathematics Education: The Role of GeoGebra CAS View and Generative Artificial Intelligence in Teaching and Learning Mathematics

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The integration of Computer Algebra Systems (CAS) in secondary mathematics education has been a subject of sustained inquiry for over four decades (Artigue, 2002). CAS tools perform symbolic manipulation—equation solving, differentiation, factorisation—enabling learners to engage with exact mathematical representations. The emergence of GeoGebra, incorporating a dedicated CAS view since version 4.2, has transformed the accessibility of symbolic computation in secondary classrooms (Hohenwarter & Preiner, 2007; Hirono & Takahashi, 2011). Concurrently, generative artificial intelligence (GenAI) tools have introduced new capabilities for mathematical explanation, problem personalisation, and natural language scaffolding (Walkington, 2025). This paper presents a systematic analysis of the role of CAS in secondary mathematics education, with attention to GeoGebra’s CAS view and its convergence with GenAI. Four research questions guide the investigation: (1) What does the literature reveal about CAS effects on teaching and learning mathematics? (2) How does GeoGebra’s multi-view architecture contribute to algebraic and calculus instruction? (3) What integration patterns emerge when GenAI is combined with CAS-based activities? (4) What pedagogical challenges and assessment implications arise from the combined use of CAS and GenAI?

Theoretical framework. The analysis draws upon three complementary perspectives. The instrumental genesis framework (Vérillon & Rabardel, 1995; Artigue, 2002; Trouche, 2004) distinguishes between an *artefact*—a material or symbolic tool—and an *instrument*, which emerges through the learner’s appropriation of the tool via instrumentalisation and instrumentation. In CAS environments, the development of CAS techniques is intrinsically linked to the development of mathematical theory (Kieran & Drijvers, 2006). When GenAI is introduced alongside CAS, a dual instrumentation challenge arises: the learner must develop schemes of use for both the CAS and the language model, and learn to coordinate them (Yumianto et al., 2024). The Technological Pedagogical Content Knowledge (TPACK) framework (Mishra & Koehler, 2006) posits that effective technology-integrated instruction requires the intersection of content, pedagogical, and technological knowledge; GenAI adds a fourth technological dimension, as teachers must understand the affordances and limitations of language models in mathematical contexts, including the skill of prompt engineering (Dertli & Yıldız, 2025). The theory of multiple representations (Duval, 1999) holds that mathematical understanding deepens when learners work across symbolic, graphical, numerical, and verbal registers; GenAI introduces natural language as a fifth representational register, though whether this additional register deepens or fragments mathematical understanding remains an open question.

GeoGebra CAS view in secondary mathematics. GeoGebra’s multi-view architecture links the CAS view, powered by the Giac kernel for exact symbolic computation, bidirectionally with the algebra, graphics, and 3D views. This linkage produces pedagogical effects that neither CAS nor dynamic geometry achieves alone (Hirono & Takahashi, 2011). The literature documents CAS-enabled activities across secondary education: linear equations and analytic geometry (Wapinda et al., 2018), quadratic functions with slider-based parameter exploration where positive effects on conceptual understanding, representational skills, and student motivation have been reported (Sun, 2023; Pospos & Piñero, 2024), polynomial inequalities where prerequisite mathematical knowledge proved critical for successful instrumentation (Tarraf et al., 2018), and calculus concepts including difference quotients, derivative tracing, and area under curves (Hohenwarter, 2008). The empirical evidence, whilst generally positive, requires cautious interpretation, as few studies isolate the CAS view from other GeoGebra views.

Generative AI and CAS convergence. The mathematical capabilities of current large language models are substantial but uneven. These systems can perform routine algebraic manipulations and generate code for mathematical software; however, their reasoning is fundamentally pattern-based rather than deductive, producing characteristic failure modes: sign errors, structural blind spots in degenerate cases, domain violations, and confident presentation of incorrect reasoning (Botana & Recio, 2024; Matzakos et al., 2023). This asymmetry—competent natural language interaction coupled with unreliable symbolic computation—defines a complementary model: GenAI is suited for natural language explanation, problem personalisation, and code scaffolding, whilst GeoGebra CAS provides reliable symbolic computation, algebraic verification, and dynamic visualisation (Botana et al., 2024). Six integration workflows are identified: (1) AI generates solutions, CAS verifies; (2) AI scaffolds GeoGebra code, students debug and extend; (3) AI personalises problems, CAS solves; (4) CAS produces results, AI explains the reasoning; (5) custom AI assistants pre-configured for specific CAS topics with Socratic questioning strategies; and (6) “teach the AI” activities verified through CAS computation (Parra et al., 2025; Ozgun-Koca et al., 2026; Xing et al., 2025).

Pedagogical challenges. Seven categories of pitfalls arise from the combined use of CAS and GenAI: (1) the prompt engineering gap, where students cannot formulate sufficiently precise prompts because effective prompting presupposes the mathematical understanding the activity is intended to develop (Yunianto et al., 2024); (2) uncritical acceptance of AI-generated mathematics, as language models present incorrect reasoning with the same linguistic confidence as correct reasoning (Parra et al., 2025); (3) prerequisite knowledge barriers that hinder CAS instrumentation (Tarraf et al., 2018); (4) AI functioning as procedural scaffolding without promoting creative mathematical reasoning; (5) over-reliance and increased cognitive load from managing two tools simultaneously; (6) inconsistency in AI outputs across sessions owing to the probabilistic nature of language model generation; and (7) CAS syntax debugging difficulties with AI-generated code (Ozgun-Koca et al., 2026). A graduated approach—establishing CAS competence before introducing AI—combined with mandatory CAS verification as an assessed step, constitutes the principal countermeasure identified in the literature.

Assessment strategies. Assessment must target mathematical reasoning rather than computational output. Drawing on two decades of CAS assessment research (Stacey, 2003; Jankvist et al., 2021), a four-dimension rubric framework is proposed: (i) mathematical correctness verified via CAS, (ii) reasoning quality, (iii) tool-use judgement, and (iv) critical evaluation of AI outputs (Wazan, 2026). An AI Audit task format operationalises this framework: students receive pre-generated AI responses (approximately 70% containing errors of varying types), compare them with CAS results, diagnose discrepancies, and explain the mathematical source of errors. This format assesses conceptual understanding, CAS fluency, and critical AI literacy simultaneously.

Conclusions and research gap. Five conclusions emerge. First, CAS shifts instruction from procedural computation towards conceptual understanding when activities promote reasoning and students possess sufficient prerequisite knowledge. Second, GeoGebra’s multi-view architecture enables distinctive multi-representational mathematical activity with positive effects on understanding and motivation across secondary education. Third, GenAI and CAS operate as complementary rather than competing tools, with six integration workflows exploiting this

complementarity. Fourth, seven categories of pedagogical challenges necessitate graduated introduction and reasoning-focused assessment. Fifth, and most significantly, no controlled classroom study has yet evaluated the combined deployment of GeoGebra CAS view and GenAI in secondary settings, constituting the principal direction for future empirical research.

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