Title: Filament structure in random plane waves

Abstract: Numerical studies of random plane waves, functions

$$u = \sum_j c_j e^{\frac{i}{h}\langle x,\xi_j\rangle}$$

where the coefficients c_j are chosen "at random", have detected an apparent filament structure. The waves appear enhanced along straight lines. There has been significant difference of opinion as to whether this structure is indeed a failure to equidistribute, numerical artefact or an illusion created by the human desire to see patterns. In this talk I will present some recent results that go some way to answering the question. First we consider the behaviour of a random variable given by $F(x,\xi) = ||u||_{L^2(\gamma_{(x,\xi)})}$ where $\gamma_{(x,\xi)}$ is a unit ray from the point x in direction ξ. We will see that this random variable is uniformly equidistributed. That is, the probability that for any (x,ξ), $F(x,\xi)$ differs from its equidistributed value is small (in fact exponentially small). This result rules out a strong scarring of random waves. However, when we look at the full phase space picture and study a random variable $G(x,\xi) = ||P_{(x,\xi)}u||_{L^2}$ where $P_{(x,\xi)}$ is a semiclassical localiser at Planck scale around (x,ξ) we do see a failure to equidistribute. This suggests that the observed filament structure is a configuration space reflection of the phase space concentrations.