Math 563, Fall 2016
Assignment 1, due Wednesday, September 7

Hand in the following exercises. Note that there is a difference between an "exercise" and a "problem" in the text, below we refer to the former.

1. Stein-Shakarchi, Exercise #2, Chapter 1. In part (d), you are being asked to extend the function by the given procedure, then prove that the extension is continuous on $[0, 1]$.

2. Stein-Shakarchi, Exercise #4(a,b), Chapter 1. We will discuss the solution to this at some point in class.

3. Stein-Shakarchi, Exercise #28, Chapter 1.

4. Let $\mathbf{a} = \{a_n\}_{n=1}^{\infty}$ denote a sequence of complex numbers. Define

$$\ell^2(\mathbb{N}) = \left\{ \mathbf{a} = \{a_n\}_{n=1}^{\infty} : \sum_{n=1}^{\infty} |a_n|^2 < \infty \right\},$$

that is, $\ell^2(\mathbb{N})$ is the collection of all complex sequences which are square summable. On your own, check that

$$d(\mathbf{a}, \mathbf{b}) = \left(\sum_{n=1}^{\infty} |a_n - b_n|^2 \right)^{1/2}$$

defines a metric on $\ell^2(\mathbb{N})$ (with $\mathbf{b} = \{b_n\}_{n=1}^{\infty}$). Hand in a proof that the metric space $(\ell^2(\mathbb{N}), d)$ is complete.

Reading: Stein and Shakarchi, Chapter 1

On your own:

1. If b is an integer larger than 1 and $0 < x < 1$, show that there exist integer coefficients c_k, with $0 \leq c_k < b$, such that $x = \sum_{k=1}^{\infty} c_k b^{-k}$. Show that this expansion is unique unless $x = cb^{-k}$ for some integer c, which case there are two expansions.

2. Find or recall the proof of the theorem stating that a bounded function $f : [a, b] \to \mathbb{R}$ is Riemann integrable if and only if its set of discontinuities has measure zero.

3. Stein-Shakarchi, Exercise #15, Chapter 1.