Math 402/502, Spring 2018
Assignment 11, due Wednesday, April 25

Problems to hand in:

1. Wade, Exercise 11.2.7.
2. Wade, Exercise 11.2.10.

   Note: Solve this exercise without using the chain rule from §11.4.
3. Wade, Exercise 11.3.6.
4. Wade, Exercise 11.4.3.

   Hint: For each $\mathbf{x}$, compute $\frac{\partial}{\partial \rho}(f(\rho \mathbf{x}))$ at $\rho = 1$ in 2 different ways.
5. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a C^2 function on $\mathbb{R}^2$ and set $u(r, \theta) = f(r\cos\theta, r\sin\theta)$.

   (a) Show that

   $$\frac{\partial u}{\partial r} = \cos\theta \frac{\partial f}{\partial x} + \sin\theta \frac{\partial f}{\partial y}$$
   $$\frac{\partial u}{\partial \theta} = -r\sin\theta \frac{\partial f}{\partial x} + r\cos\theta \frac{\partial f}{\partial y}$$

   (b) Show that

   $$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

On your own: Wade 11.2.3, 11.2.6, 11.2.8, 11.2.9[1], 11.3.2, 11.3.5, 11.4.2, 11.4.4, 11.4.5, 11.4.8.
Reading: Wade, sections 11.3, 11.4, 11.5.

[1] Using this exercise and the arithmetic-geometric mean inequality $|xy| \leq \frac{1}{2}(x^2 + y^2)$, you can make quick work of 11.2.6, so go for it!