Math 402, Spring 2018
Assignment 1, due Wednesday, January 24

Problems to hand in

1. Suppose $x, y \in \mathbb{R}$. Use a proof by contrapositive to show the following statement:

   If $x \leq y + \epsilon$ for every $\epsilon > 0$, then $x \leq y$.

   Note: the contrapositive isn't hard to show here, but much like the "on your own" problems below, this principle comes up **frequently**.

2. Suppose $A \subset \mathbb{R}$ is a nonempty set which is bounded from above and below. Prove that

   $$\sup A - \inf A = \sup\{|a_1 - a_2| : a_1, a_2 \in A\} = \sup\{a_1 - a_2 : a_1, a_2 \in A\}.$$

3. Wade, Exercise 1.3.7.

4. Wade, Exercise 5.1.8.

   Hint: Begin by justifying that $M_j(f) - m_j(f) = f(x_j) - f(x_{j-1})$ using Exercise 1.3.7a.

On your own (i.e. do not hand these in for a grade)

Solve the problems in Wade: 1.3.0, 1.3.6, 1.3.8, 5.1.1, 5.1.2b, as well as the following problems:

1. Suppose $a, b, c, d \in \mathbb{R}$ and that $a \leq b \leq c \leq d$. Show that $c - b \leq d - a$.

   Note: This is not hard, but this principle comes up frequently, so it is worthwhile to be very familiar with it.

2. Suppose $X \subset \mathbb{R}$ and $f : X \to \mathbb{R}$. Given a nonempty set $E \subset X$, use Problem 2 above to show that

   $$\sup_{t \in E} f(t) - \inf_{t \in E} f(t) = \sup\{|f(t) - f(s)| : t, s \in E\} = \sup\{f(t) - f(s) : t, s \in E\}.$$

   Note: This is nearly immediate from Problem 2, but its importance is due to its use in integration theory. For example, if $X = [a, b]$ and $E = [x_{j-1}, x_j] \subset [a, b]$ is a subinterval determined by a partition, then this says that

   $$\begin{aligned} M_j(f) - m_j(f) &= \sup\{|f(t) - f(s)| : t, s \in [x_{j-1}, x_j]\} \\ &= \sup\{f(t) - f(s) : t, s \in [x_{j-1}, x_j]\}. \end{aligned}$$

Reading and review

Wade, Sections 1.3, 5.1. Review supremum and infimum.