Math 402/502, Spring 2018
Assignment 4, due Wednesday, February 14

Problems to hand in:

1. Decide if the following series are convergent or divergent. Use any test you like, but fully verify its hypotheses.

   (a) $\sum_{k=1}^{\infty} \frac{k^2-3k+1}{3k^2+k-2}$.

   (b) $\sum_{k=1}^{\infty} \frac{\sqrt{k}}{k^2-k+2}$.

   (c) $\sum_{k=1}^{\infty} \frac{1}{\log(1+k)}$.
   Hint: Begin by showing $\log(1+k) \leq k$ for sufficiently large k. L'Hôpital's rule provides one way to accomplish this.

   (d) $\sum_{k=1}^{\infty} \frac{1}{k \log^p k}$, where $p \leq 1$.

   (e) $\sum_{k=1}^{\infty} \frac{1}{(3+(-1)^k)^k}$.

   (f) $\sum_{k=1}^{\infty} \frac{k2^k}{3^k}$.

2. Suppose that $\{a_k\}_{k=1}^{\infty}$ is a decreasing sequence of positive real numbers. Prove that if $\sum_{k=1}^{\infty} a_k$ converges, then $ka_k \to 0$ as $k \to \infty$.

   Hint: Start by observing that

   $$na_{2n} \leq \sum_{k=n}^{\infty} a_k, \qquad \text{and} \qquad na_{2n+1} \leq \sum_{k=n}^{\infty} a_k.$$

3. Wade, Exercise 6.1.6.

4. Wade, Exercise 6.2.6.

5. Wade, Exercise 6.3.7.

On your own (i.e. do not hand these in for a grade): Wade 6.1.1, 6.1.3, 6.1.4[1], 6.1.5, 6.2.1, 6.2.2, 6.2.3, 6.2.5, 6.3.1, 6.3.2, 6.3.3(a-e), 6.3.4. as well as the following problem:

Let $s_n := 1 + \frac{1}{2} + \cdots + \frac{1}{n}$. Prove that $s_{2k} - s_k > \frac{1}{2}$ and use this to give another proof that the harmonic series $\sum_{k=1}^{\infty} \frac{1}{k}$ diverges.

Reading: Wade, sections 6.1-6.4.

[1]This isn't quite as hard as it looks if you write $a_{k+1} - 2a_k + a_{k-1} = (a_{k+1} - a_k) - (a_k - a_{k-1})$.